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WIND TURBINE DIRECTIONAL TONALITY

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ABSTRACT

Knowledge of the emission patterns (directivity) of modern wind turbines improves the prediction of noise levels at receptors over medium and long distance. Tonality is usualy attributed as a penalty or character correction factor in most regulations. In some of them, the penalty, up to 6 dB, is equivalent to quadrupling the power of the wind turbine. To predict the impact caused to residents near wind farms, it is desirable to know precisely both the directivity of the wind turbines and the potential tonality.

The present work shows the results of the tonality around a wind turbine and not only for the downwind conditions of the IEC standard. It was carried out for different wind speeds. The aim of this work is to contribute to the improvement of the prediction of wind farm nuisance.

Keywords: *Directivity, Tonality, Wind, Turbine*

1. INTRODUCTION

Part 11 of the IEC 61400 standard [1] outlines a structured methodology designed to ensure consistent and precise measurement and analysis of acoustic emissions from wind turbines. This section of the standard offers procedures for characterizing the noise emissions of a wind turbine by considering it as a point source situated at the rotor center.
Standard measurements are typically conducted at a designated reference location to determine the turbine's acoustic profile. However, to gain further insight into the directional tonality of emissions, supplementary measurement positions can be used. These additional measurement locations, while not commonly applied in traditional IEC testing protocols, provide valuable data concerning the directional characteristics of wind turbine noise.
By incorporating a greater number of measurement positions, this paper seeks to explore the tonal directivity of a wind turbine, offering a comprehensive analysis that extends beyond standard practices.

2. METHODOLOGY

The reference position for the measurements is specified as the downwind location, situated at a distance equal to the hub height plus rotor radius from the rotor center. As optional measurement locations, positions at upwind and downwind angles of ±60° are also considered at the same distance.

In both the downwind and upwind positions, similar sound pressure levels are anticipated, whereas greater variations are expected at the crosswind positions, as suggested by aerodynamic sound theories and empirical studies [2-5]. The directionality of the predominant aerodynamic sound source is primarily attributed to trailing edge noise; however, variations in mechanical and electrical noise can arise depending on the wind turbine's cooling system configuration or whether the power electronics are housed inside or outside the tower. Some setups may also include additional cooling systems. To evaluate the impact of these components, supplementary measurement locations may be necessary [6].

This setup aims to provide a comprehensive array of measurement points to capture a detailed acoustic profile of the wind turbine from multiple directional perspectives. By

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using these positions, both aerodynamic and mechanical noise characteristics can be thoroughly analyzed.

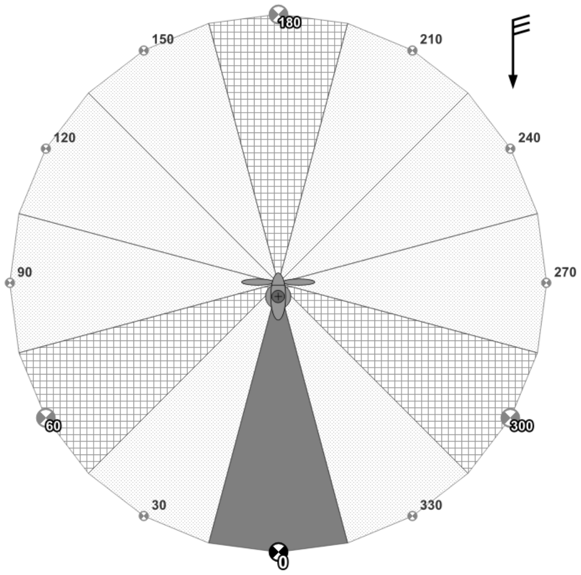

Figure 1. Plan Standard pattern for microphone measurement positions (plan view). Measurement positions for reference (0º), optional locations (60º, 180º and 300º) and complementary locations in 30º sectors.

3. TONAL AUDIBILITY

The IEC 61400-11 standard provides clear definitions and descriptions for terms related to the tonality and tonal audibility of the noise generated by wind turbines. Specifically, Section 9.5 of the standard outlines the procedure for assessing tonality across varying wind speeds through narrowband analysis.

According to this procedure, the tonality should be represented for each wind speed bin, using increments of 0.5 m/s. This approach ensures a detailed examination of tonal characteristics over a spectrum of operating conditions, thereby providing a comprehensive understanding of the noise emissions from wind turbines.

3.1 Fixed positions with board positioned on the ground

Measurements are conducted using a microphone positioned on a measurement board placed at ground level. This setup is designed to reduce wind noise impacting the microphone and minimize the influence of varying ground types on acoustic data. All measurements are synchronized with data from both the turbine and mast anemometers to ensure accurate correlation. The reference wind speed for analysis is derived from the turbine's electrical power output.

For each wind speed bin, ten measurements were selected for analysis. In this study, a sampling frequency of 25,600 samples per second was employed, resulting in data matrices with dimensions of 256,000 x 10 cells for each set of conditions. Although the wind speeds varied from 6 to 13 m/s, the results presented will focus specifically on the wind speed bins of 6 and 11 m/s.

The algorithm outlined by the standard is straightforward, though it allows flexibility in narrowband spectral resolution, accepting values between 1 and 2 Hz. The results obtained using both resolutions are practically identical [7]. In the analysis, ten measurement records each lasting 10 seconds are utilized for a given wind speed. The overall tonal audibility is confirmed if at least six out of ten narrowband spectra exhibit an identified tone originating from the same source. Tones identified in different spectra are considered to have the same origin if they fall within an interval of ±25% of the critical band centered at their frequency. Tones of the same origin are treated and reported as one tone.

Tables 1 to 19 detail the methodology employed by the implemented program in Matlab [8] for tone detection and audibility analysis. Utilizing the Fast Fourier Transform (FFT) of the ten input signals, the program computes sound levels, energy, and bandwidth for resolutions of 1 and 2 Hz. It identifies local maxima of the signals but selects only those whose levels surpass the critical band average by more than 6 dB (excluding the maximum line and its two adjacent ones) as potential tones. The *findTones()* function then locates these tones.

The signal lines are categorized as 'masking,' 'tones,' or 'neither' using the L_{70} and L_{pn} criteria (sound pressure level of masking noise within a critical band). The process concludes with the determination of tonality (the difference between the tone level and the masking noise level) and audibility, which includes a frequency-dependent correction to account for the human ear's response to tones at varying frequencies.

Table 1. Tone identification for 60° position and wind speed of 11 m/s with 1 Hz resolution.

f_c,[1] Hz	Identification of spectra with a tone at this line									
112	0	0	1	1	0	0	0	0	0	0
113	0	1	0	0	1	1	1	1	0	0
115	1	0	0	0	0	0	0	0	1	0
116	0	0	0	0	0	0	0	0	0	1
223	0	0	0	1	0	0	0	0	0	0
224	0	0	1	0	0	0	0	0	0	0
225	0	1	0	0	1	0	0	0	0	0
227	0	0	0	0	0	1	1	0	0	0
229	0	0	0	0	0	0	0	0	1	0
230	1	0	0	0	0	0	0	1	0	0
231	0	0	0	0	0	0	0	0	0	1

Table 2. Tone identification for 60° position and wind speed of 11 m/s with 2 Hz resolution.

f_c, Hz	Identification of spectra with a tone at this line									
112	0	0	1	1	0	0	0	0	0	0
114	0	1	0	0	1	1	1	1	0	0
116	1	0	0	0	0	0	0	0	1	1
224	0	0	1	1	0	0	0	0	0	0
226	0	1	0	0	1	0	0	0	0	0
228	0	0	0	0	0	1	1	0	0	0
230	1	0	0	0	0	0	0	1	1	0
232	0	0	0	0	0	0	0	0	0	1

Only tones that are present in at least six of the ten analyzed spectra and have an average audibility, $\Delta L_{a,j,k}$,[2] of equal to or greater than -3 dB are presented. A tone is considered audible if its tonal audibility exceeds 0 dB.

Table 3. Average tone and average audibility for each register, tone at 113 Hz, 1 Hz resolution.

f_c Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	fc in 10	Tone Y/N
112	0,44	2,45	10	2	Y
113	**0,44**	**2,45**	**10**	**5**	**Y**
115	0,44	2,45	10	2	Y
116	0,44	2,45	10	1	Y

Table 4. Average tone and average audibility for each register, tone at 227 Hz, 1 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	f_c in 10	Tone Y/N
223	-3,53	-1,47	10	1	Y
224	-3,53	-1,47	10	1	Y
225	-3,53	-1,47	10	2	Y
227	**-3,53**	**-1,47**	**10**	**2**	**Y**
229	-3,53	-1,47	10	1	Y
230	-3,53	-1,47	10	2	Y
231	-3,53	-1,47	10	1	Y

Table 5. Average tone and average audibility for each register, tone at f_c = 114 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,11}$	Aver Audib. (dB) $\Delta L_{a,11}$	N in 10	f_c in 10	Tone Y/N
112	1,62	3,63	10	2	Y
114	**1,62**	**3,63**	**10**	**5**	**Y**
116	1,62	3,63	10	3	Y

Table 6. Average tone and average audibility for each register, tone at 228 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	f_c in 10	Tone Y/N
224	-3,02	-0,97	10	2	Y
226	-3,02	-0,97	10	2	Y
228	**-3,02**	**-0,97**	**10**	**2**	**Y**
230	-3,02	-0,97	10	3	Y
232	-3,02	-0,97	10	1	Y

At this wind speed, 11 m/s, only one audible tone is identified at a frequency of 113 Hz (or 114 Hz when using a 2 Hz resolution). Reporting these frequencies is justified, as they correspond to the maximum number of records with a tone present at that frequency, totaling five for both scenarios.

Furthermore, a secondary tonal presence at 227 Hz aligns with the frequency of 2 * f_c, suggesting a mechanical origin, likely related to the slow axis of the nacelle.

[1] f_c centre frequency of critical band

[2] $\Delta L_{a,k}$ difference between the tonality and the audibility criterion in each wind speed bin, where k is the centre value of the wind speed bin

Table 7. Tone identification for 300° position and wind speed of 11 m/s with 1 Hz resolution.

f_c, Hz	Identification of spectra with a tone at this line									
110	0	0	0	0	0	0	0	0	0	1
111	0	0	0	0	0	0	0	1	1	0
115	1	0	0	1	0	1	1	0	0	0
116	0	1	0	0	1	0	0	0	0	0
117	0	0	1	0	0	0	0	0	0	0
5179	0	0	0	0	0	0	0	1	0	0
5181	0	0	0	0	0	0	0	0	1	0
5196	0	0	0	0	0	1	0	0	0	0
5216	0	0	0	1	0	0	0	0	0	0
5229	0	0	0	0	0	0	1	0	0	0
5404	0	0	0	0	0	0	0	0	0	1

Table 8. Tone identification for 300° position and wind speed of 11 m/s with 2 Hz resolution.

f_c, Hz	Identification of spectra with a tone at this line									
110	0	0	0	0	0	0	0	0	0	1
112	0	0	0	0	0	0	0	1	1	0
114	1	0	0	0	0	0	0	0	0	0
116	0	1	0	1	1	1	1	0	0	0
118	0	0	1	0	0	0	0	0	0	0
5180	0	0	0	0	0	0	0	1	0	0
5182	0	0	0	0	0	0	0	0	1	0
5196	0	0	0	0	0	1	0	0	0	0
5216	0	0	0	1	0	0	0	0	0	0
5230	0	0	0	0	0	0	1	0	0	0
5404	0	0	0	0	0	0	0	0	0	1

Table 9. Average tone and average audibility for each register, tone at 115 Hz, 1 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	f_c in 10	Tone Y/N
110	2,75	4,76	10	1	Y
111	2,75	4,76	10	2	Y
115	**2,75**	**4,76**	**10**	**4**	**Y**
116	2,75	4,76	10	2	Y
117	2,75	4,76	10	1	Y

Table 10. Average tone and average audibility for each register, tone can`t be clearly locate, 1 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	f_c in 10	Tone Y/N
5179	-3,53	-1,47	6	1	Y
5181	-3,53	-1,47	6	1	Y
5196	-3,53	-1,47	6	1	Y
5216	-3,53	-1,47	6	1	Y
5229	-3,53	-1,47	6	1	Y
5404	-3,53	-1,47	7	1	Y

Table 11. Average tone and average audibility for each register, tone at 113 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,11}$	Aver Audib. (dB) $\Delta L_{a,11}$	N in 10	f_c in 10	Tone Y/N
110	2,69	4,70	10	1	Y
112	2,69	4,70	10	2	Y
114	2,69	4,70	10	1	Y
116	**2,69**	**4,70**	**10**	**5**	**Y**
118	2,69	4,70	10	1	Y

Table 12. Average tone and average audibility for each register, tone at 228 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,11}$	Aver Audib. (dB) $\Delta L_{a,j,11}$	N in 10	f_c in 10	Tone Y/N
5180	-4,78	-0,24	6	1	Y
5182	-4,78	-0,24	6	1	Y
5196	-4,78	-0,24	6	1	Y
5216	-4,78	-0,24	7	1	Y
5230	-4,78	-0,24	7	1	Y
5404	-4,78	-0,24	7	1	Y

The tonal noise identified in the high-frequency range is attributed to background noise from bird songs and is not considered a tonal component originating from the wind turbine. To accurately assess this background noise, narrowband spectra should be utilized to isolate and analyze these frequencies.

Table 13. Tone identification for 60° position and wind speed of 6 m/s with 2 Hz resolution.

f_c, Hz	Identification of spectra with a tone at this line									
220	0	0	1	1	1	0	0	0	0	0
222	0	0	0	0	0	1	0	0	0	0
224	0	1	0	0	0		1	0	0	0
226	1	0	0	0	0	0	0	0	0	0
242	0	0	0	0	0	0	0	0	1	0
246	0	0	0	0	0	0	0	0	0	1

Two different critical bands are identified very close one to each other. One with centre frequency at 220 and the other at around 250 Hz.

Table 14. Average tone and average audibility for each register, tone at 220 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,6}$	Aver Audib. (dB) $\Delta L_{a,j,6}$	N in 10	f_c in 10	Tone Y/N
220	0,34	2,4	9	3	Y
222	0,34	2,4	9	1	Y
224	0,34	2,4	9	2	Y
226	0,34	2,4	9	1	Y

For low wind speed only at 60 degree position two critical band are obtained, at 220 Hz centre frequency.

Table 15. Average tone and average audibility for each register, tone at 242-246 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,6}$	Aver Audib. (dB) $\Delta L_{a,j,6}$	N in 10	f_c in 10	Tone Y/N
242	0,31	2,36	10	1	Y
246	0,31	2,36	10	1	Y

The highest tone audibility is found in the range of 220 Hz and 246 Hz, with $\Delta L_{a,6}$ ==2.4 dB. This results is lower than the maximum tone at 11 m/s $L_{a,11}$=3.63 dB at f_c=114 Hz.

Table 16. Tone identification for 300° position and wind speed of 6 m/s with 2 Hz resolution.

f_c, Hz	Identification of spectra with a tone at this line									
140	0	0	0	0	0	1	1	1	0	0
144	0	0	0	0	1	0	0	0	0	0
154	0	0	1	0	0	0	0	0	0	0
164	1	0	0	0	0	0	0	0	0	0
224	0	0	0	1	0	0	0	0	0	0
228	0	0	0	0	0	0	0	0	0	0
232	0	0	1	0	0	0	0	0	0	1
250	1	1	0	0	0	1	1	1	1	0

In the case of 300º position three different critical bands are identified.

Table 17. Average tone and average audibility for each register, tone around 150 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,6}$	Aver Audib. (dB) $\Delta L_{a,j,6}$	N in 10	f_c in 10	Tone Y/N
140	-0,94	1,08	6	3	Y
144	-0,94	1,08	6	1	Y
154	-0,94	1,08	6	1	Y
164	-0,94	1,08	6	1	Y

Table 18. Average tone and average audibility for each register, tone around 228 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,6}$	Aver Audib. (dB) $\Delta L_{a,j,6}$	N in 10	f_c in 10	Tone Y/N
224	2,21	4,28	10	1	Y
228	2,21	4,28	10	1	Y
232	2,21	4,28	10	1	Y

Table 19. Average tone and average audibility for each register, tone around 250 Hz, 2 Hz resolution.

f_c, Hz	Aver. Tonal (dB) $\Delta L_{tn,j,6}$	Aver Audib. (dB) $\Delta L_{a,j,6}$	N in 10	f_c in 10	Tone Y/N
250	**2,29**	**4,36**	**9**	**6**	**Y**

The highest average audibility for low wind, 6 m/s at hub height, is found at f_c=250 Hz with $\Delta L_{a,6}$ =4.36 dB, similar to the audibility of $\Delta L_{a,11}$=4.76 dB resulted at 11 m/s for the same position but at different central frequency f_c=114 Hz.

3.2 Short time complementary positions

Short-time measurements were conducted around the wind turbine. All measurements were performed using a tripod set at a height of 1.5 meters. Events not associated with the wind turbine source under investigation are excluded from the analysis.

Measurements are carried out with an emphasis on minimizing wind noise at the microphone. A temporary mast-mounted anemometer serves as a reference to ensure accuracy, with measurements discarded if wind speeds exceed 5 m/s at 3 meters above ground level. Consistent with the IEC standard, the reference wind speed is derived from the turbine's electrical power output.

In parallel with the IEC methodology test, additional logs were obtained using a moving tripod, maintaining a distance of hub height plus rotor radius. The testing was extended up to the 180-degree position (upwind) in 30-degree increments relative to the yaw orientation, ranging from 0 degrees (downwind) to 180 degrees (upwind). These measurements incorporated A-weighted spectra, including FFT analysis with a sound level meter integrated at a 1.465 Hz resolution, alongside third-octave band recordings. These positions were synchronized with recordings from the fixed ground-mounted board and turbine data to ensure comprehensive analysis.

The reference condition for measurements is established during middle-range turbine operation, which corresponds to 30% to 60% of the rated power. This operational range represents wind speeds at hub height between 6 and 11 m/s, effectively minimizing background noise levels and wind-induced distortion. All measurements are conducted under conditions where wind speeds remain below 5 m/s, as monitored by a temporary meteorological pole synchronized with the system.

Short-time complementary measurement positions focus on the frequency range of 20 Hz to 1500 Hz. These efforts aim to minimize background noise effects while identifying frequencies detectable from standard positions.

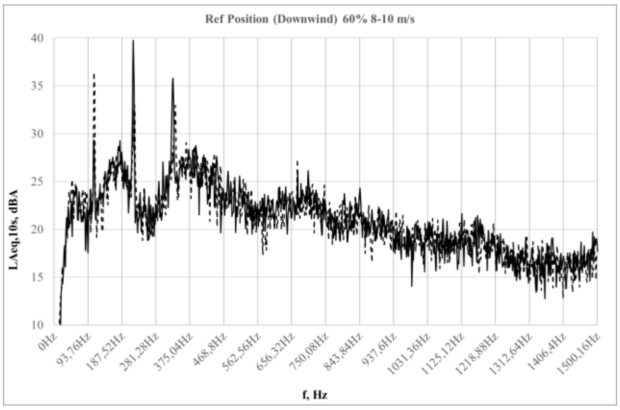

Figure 2. A-weighted FFT-spectra for reference position at 60% of capacity.

The FFT-spectra analysis reveals similar tonal components to those identified in the board-positioned measurements, specifically at frequencies of 110 Hz, 218 Hz, and 328 Hz. These complementary positions are strategically employed to corroborate findings from automated audio extraction analysis, ensuring consistency and accuracy in the detection and characterization of tonal components.

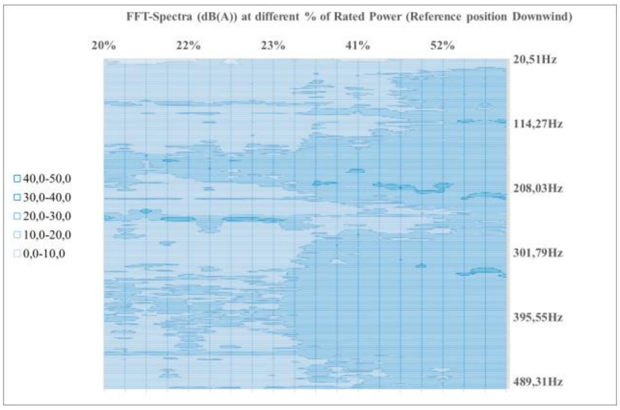

Figure 3. FFT-spectra change at different operational conditions 20 to 60% of active power output at downwind reference position.

Selected spectra from the same location are grouped and organized according to output power and derived wind speed. In Figure 4, the tones identified per the IEC standard exhibit a frequency shift from f_c = 83 Hz to 114 Hz, emerging at higher power ratings. Additionally, this shift reveals three harmonics at frequencies of 2 * f_c and 3 * f_c.

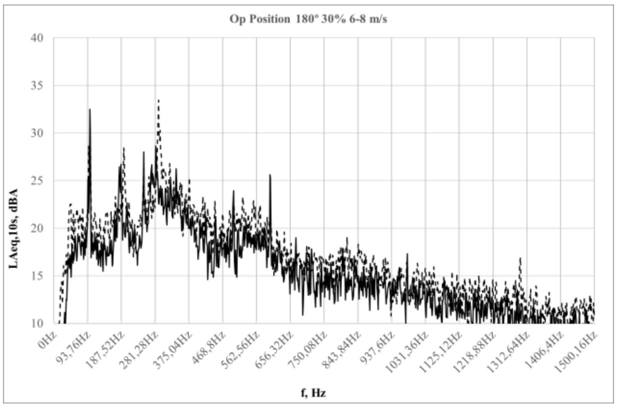

Figure 4. A-weighted spectra for upwind position at 30% of capacity.

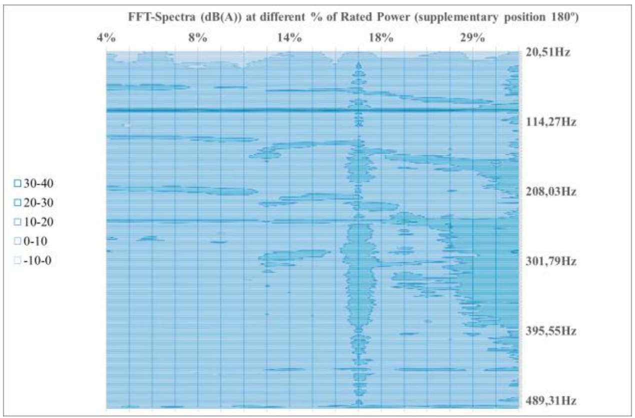

Figure 5. FFT-spectra change at different operational conditions 5 to 30% of active power output at 180 position (upwind).

The results from the upwind position indicate an increase in the tonal component at 100 Hz. The origin of this tone is electrical and aligns with the behavior typically observed in transformers. The tonality at 100 Hz remains stable across all operational rates, whereas the predominant tone at 113 Hz, identified at the IEC-positioned measurements, varies according to the wind turbine's rotary speed.

The sources and locations of the two tonal components are distinct. However, under specific operational conditions, they can converge at the same frequency, leading to enhanced audibility and perception of the tones.

4. CONCLUSIONS

The IEC 61400-11 standard specifies the measurement position to be in the downwind condition. It assumes an apparent sound power level from a point source located at the rotor center, with emissions in the downwind direction analogous to those of the wind turbine.

Experimental noise measurements around a modern wind turbine have been carried out to explore the tonal directivity associated with the turbine.

The primary noise source of a modern wind turbine is attributed to the blades. Specifically, it is recognized that the main emissions originate from the last third of the blade. Additionally, the trailing edge noise exhibits a non-omnidirectional directivity, varying with different wind speeds, rotor speeds, and pitch angles.

Mechanical and electrical components can contribute to the tonal characteristics of a wind turbine. These components tend to be more audible at lower wind speeds, as emissions from the rotor decrease and background noise is minimal.

The noise emitted by the wind turbine was measured in the absence of other external sources such as other wind turbines, industrial noise, or traffic noise. Some bird songs have been identified in the recordings, contributing primarily in the 4-5 kHz range. These contributions do not affect the results, which focus on the frequency range of 20 to 1500 Hz.

The automated audio analysis from fixed positions (with board positioned on the ground) shows good correlation with the short term moving positions (tripod at 1.5 meter).

Ideally, reference measurements and all complementary measurements should be performed simultaneously, adhering to the methodology outlined in the IEC standard. Utilizing synchronized parallel measurements conducted over short periods at 1.5 meters above ground level demonstrates excellent correlation, particularly when the maximum wind speed is controlled and limited. The ground effect, in general, must be considered and corrected to ensure measurement accuracy.

This technique offers increased flexibility while maintaining reference positions downwind, enabling the investigation of tonal components whose origins may not be readily apparent from a fixed position.

Certain components of wind turbines can contribute to overall sound levels and, particularly, to tonal components, which may not originate from the hub height. These components may have directivity characteristics that might alter test results, as they deviate from the assumption that emissions are centered at the rotor location.

When noise sources are situated close to ground level, the topography of the terrain significantly influences sound propagation. The selection of test conditions, including the IEC-specified downwind positions, can alter results, especially concerning components close to ground level.

The use of optional and complementary measurement positions reveals significant differences in audibility assessments. These variations can even alter the dominant tonal frequency observed in the analysis.

Further investigations should involve additional measurements across different turbine types. Measurements at higher wind speeds are preferable, with devices positioned on the ground, and simultaneous assessments conducted in various optional or complementary locations. The variability in wind direction presents a challenge for these tests. Effective synchronization of non-acoustic data aids in filtering results when necessary. However, ideally, stable conditions are preferred, allowing measurements to cover different wind speeds within a shorter period of time and enabling precise repositioning.

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