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EXPERIMENTAL MEASUREMENTS OF THE DIFFUSION COEFFICIENT FOR THE DIFFUSION EQUATION OF THE SCHOENENBOURG FORT TUNNELS

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Abstract

The diffusion equation model has been used for room acoustics simulations because of its computational efficiency. Traditionally, the diffusion coefficient of the diffusion equation has been treated as a constant depending on the room's volume and total surface area. However, numerical investigations have demonstrated that in elongated spaces, this assumption does not hold: the diffusion coefficient varies with spatial location, source position, and absorption coefficient of the boundaries. Despite these theoretical insights, experimental validation of spatial dependencies is lacking. This study addresses this gap by presenting experimental measurements of the diffusion coefficient within one of the tunnels of the Schoenenbourg fort, a structure belonging to the historic Maginot line in France. The methodology relies on intensity and energy density measurements taken along the tunnel, allowing the diffusion coefficient to be derived directly from its definition given by Fick's law. The measurement results demonstrate that the diffusion coefficient cannot be regarded as a constant in elongated spaces but instead depends on spatial conditions. In addition, a preliminary validation was performed by incorporating the experimentally obtained diffusion coefficient into the diffusion equation model. The comparison revealed that the model, when used with the measured diffusion coefficient, successfully reproduces the tunnel sound pressure level and reverberation time.

Keywords: *room acoustics, diffusion equation, long rooms, acoustics measurements*

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1. Introduction

In recent decades, the diffusion equation has emerged as a modeling method for predicting room acoustics [1]. This method is based on the analogy with diffusion processes in heat conduction and fluid dynamics, in which the field is described by particles moving within a scattering medium. In the acoustic analogy, sound energy is represented by particles propagating through air and scattering at the room boundaries [2], [3]. Under the assumption of isotropic behaviour of sound and Fick's law of diffusion, the time-dependent diffusion equation model is derived for the acoustic energy density $w(\mathbf{r},t)$ in [kg / (m s^2)] [4] together with its boundary conditions:

$$\begin{cases} \dfrac{\partial w(\mathbf{r},t)}{\partial t} = -\nabla \cdot (D(\mathbf{r})\nabla w(\mathbf{r},t)) + q(\mathbf{r},t) \\ -D(\mathbf{r})\dfrac{\partial w}{\partial n} = cA_x w \end{cases} \tag{1}$$

where $\mathbf{r} = (x, y, z)$ is the position vector, c is the speed of sound, q is the energy per volume of an omnidirectional sound source, n is the normal to the surface, and A_x is the modified absorption factor. The modified absorption factor depends on the absorption coefficient of the particular surface and is defined as $\dfrac{\alpha}{2(2-\alpha)}$ [5]. The diffusion coefficient $D(\mathbf{r})$ is a proportionality factor that scales the energy density of the room. It is generally treated as a constant value based on the geometry of the space [4]:

$$D_{\mathrm{th}} = \frac{\lambda c}{3}, \tag{2}$$

where D_{th} stands for the theoretical diffusion coefficient. Within classical acoustic theory, λ is the mean free path of the room in [m] and is more commonly expressed as $4V/S$, with V corresponding to the volume of the room and S to its entire boundary surface area.

The diffusion coefficient, as described above, is generally accurate for proportional rooms with low absorption surfaces. However, for non-proportional rooms, its accuracy decreases significantly [6]. Over the years, numerous researchers have attempted to better understand the diffusion coefficient for non-proportional long rooms. By numerically calculating the sound field, specifically the sound intensity vector I and the gradient of the reverberant energy density $w(\mathbf{r})$ (in kg / (m s^2)), the researchers have estimated the diffusion coefficient along the main longitudinal x-axis [7]. This estimate is based on Fick's law of diffusion, assuming a constant intensity and energy density across the room's cross-section and a main power flow along the x-axis.

$$D_x = -\frac{I_x}{\frac{\partial w}{\partial x}} \tag{3}$$

The diffusion coefficient can be determined using Equation (3), but requires knowledge of the intensity of the sound and the energy density. Subsequently, the nature of the diffusion coefficient has been investigated through simulations, optimisation techniques, and measurements on scale models for long rooms [8], [9]. However, these attempts focused mainly on adjusting or interpreting the diffusion coefficient indirectly to predict the sound pressure level (SPL) and the reverberation time (T_{20}). Direct measurements of the diffusion coefficient in real-scale spaces remain limited.

The present study addresses this gap by directly determining the diffusion coefficient, using Equation (3), from spatial measurements of the sound intensity and energy density. Acoustic measurements are carried out in a real long exceptional space, the Schoenenbourg Fort tunnel, allowing the diffusion coefficient to be quantified experimentally. The measured diffusion coefficient is validated by inserting it into the diffusion equation model and comparing the resulting SPL and T_{20} prediction with experimental measurements.

2. Site descriptions

The Schoenenburg fort represents one of the most prominent sections along the Maginot Line, a series of fortifications constructed before World War II to defend against hostile attacks. Situated approximately 50 km north of Strasbourg, this underground fortress extends about 30 m below the ground level and spans approximately 1500 m in length. The fort comprises interconnected tunnels and surface-level blocks, each designed for specific military purposes, including infantry casemates, retractable gun turrets, artillery blocks, ammunition entrances, and personnel entrances. All blocks are connected through an intricate network of underground passageways [10]. A schematic layout of the Schoenenbourg fort is shown in Figure 1.

From this study, a tunnel approximately 300 m long (with a 112.5 m segment specifically used for the measurements) was selected for the investigation. This tunnel is marked in Figure 1.

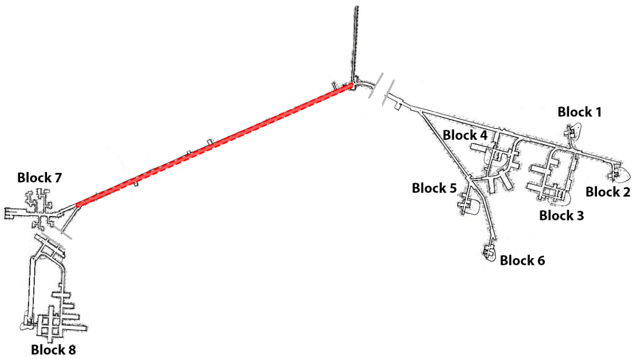

Figure 1: Schoenenbourg fort plan [10].

Figure 2: 300 m long tunnel.

The tunnel, illustrated in Figure 2, spans 2.5 m in width and features an arched ceiling that rises from 2 m on the side walls to a maximum of 3 m at its highest point. The lateral long surfaces, it presents some recesses for the passage of cables between 0.85 m and 2 m from the floor. The 112.5 m segment of the tunnel is assumed to be fully open at its extremities, as the potential return energy is negligible.

The concrete structure makes the tunnel acoustically reflective, but not fully diffusely scattering, because of the smoothness of the boundary surfaces.

Figure 3: Schematic representation of the section of the tunnel with measurement set-up and positions. The red dot indicates the source position, the blue dots indicate the impulse response measurement positions, and the green dots indicate the intensity measurement positions.

3. Acoustics measurements

Acoustic measurements were performed on the nights between March 19th and 21st, 2025, from 6:00 pm to 2:00 am, when the facility was closed to the public, ensuring minimal background noise and disturbance. The air conditioning system was turned off to eliminate mechanical noise. The experiments were conducted under controlled environmental conditions of 13 degrees and humidity 50%. Two measurement campaigns were conducted. Campaign A consisted of impulse response measurements to obtain reverberation time over the length of the tunnel. Campaign B consisted of intensimetric measurements to simultaneously acquire sound pressure and intensity data, which were then used to calculate the diffusion coefficient of the diffusion equation via Equation (3). An omnidirectional sound source (Bruel & Kjaer 4296) connected to the Lab Gruppen type 300 power amplifier was used in both campaigns. The excitation signals were generated using DIRAC 6 software running on a personal computer equipped with a Triton Acoustics Engineering sound card. Two types of signal were emitted: a 10.6 s exponential sine sweep for Campaign A and a 10.6 s maximum-length sequence (MLS) for Campaign B. Each signal was repeated twice, with a frequency range from 31.5 Hz to 16000 Hz and a sampling frequency of 48000 Hz. The sound source was positioned 100 m from the end of the tunnel (100 m, 1.25 m, 1.5 m). For Campaign A, two preamplified microphones (Bruel & Kjaer 4189 and Bruel & Kjaer 2671) were used, connected to a Triton sound card. One microphone was fixed at a distance of 2 m from the source at coordinates (102 m, 1.25 m, 1.5 m) to allow for any fluctuations to be observed, while the second microphone was moved along the tunnels in 2 m increments, resulting in a total of 56 measurement positions (R01-R56). For campaign B, an intensity probe (Bruel & Kjaer 4197) connected to a sound level meter (Bruel & Kjaer 2270) was used. The system was calibrated before and after measurements, both for level and phase. The intensity probe was moved along the x-axis of the tunnel in increments of 0.5 m, yielding 225 measurement positions (I001-I225). Assuming a constant diffusion coefficient and sound pressure level in the cross section of the tunnel, only the intensity along the x-axis was measured. The equipment information used for the measurements is available upon request. The measurement layout is shown in Figure 3.

The INR, a measure of the quality of the impulse response [11], was checked to be greater than 35 dB for most frequencies; therefore, the quality of the impulse response and the estimation of the reverberation time can be considered satisfactory [12]. In addition, the background level was checked to be at least 10 dB lower than the average level.

4. Measurement results

In this section, intensity measurement results of the long tunnel are discussed, only for the 1000 Hz frequency band. Similar results were found for other frequency bands. The output of the intensity sound meter is presented in terms of intensity in [W/m^2] and energy density in [kg/m s^2] in Figure 4(a, b).

Figures 4 (a, b) show that the intensity and energy density over the distance between source and receiver along the x-axis decrease with an exponential decay. To enable the calculation of the diffusion coefficient, the raw values of intensity and energy density are fitted using a single-term exponential function:

$$y_j = a_j e^{b_j x} \tag{4}$$

where y_j is the fitted value of intensity or energy density with j being $= [I, w]$, x is the distance between source and receiver, and a_j, b_j are the fitted coefficients for respectively intensity I and energy density w. The starting point of the fitting is chosen after two mean free path distances from the source, as the values close to the source are not accurate enough when calculated with the diffusion equation, since the sound field close to the source is dominated by a direct field and not by a diffusion process [13]. From these fitted values, the gradient of the energy density of Figure 4 (c) is calculated using a finite difference scheme and the diffusion coefficient Figure 4 (d) is subsequently determined using Equation (3).

As shown in Figure 4, the gradient of the energy density increases along the x-axis. The obtained diffusion coefficient is not constant and increases almost linearly with distance from the source. This agrees with the findings of [7], [8], [9] based on numerical simulations and scale model experiments. Increasing the frequency yields smaller values of the derived diffusion coefficient and a smaller gradient in the slope of the diffusion coefficient with respect to distance.

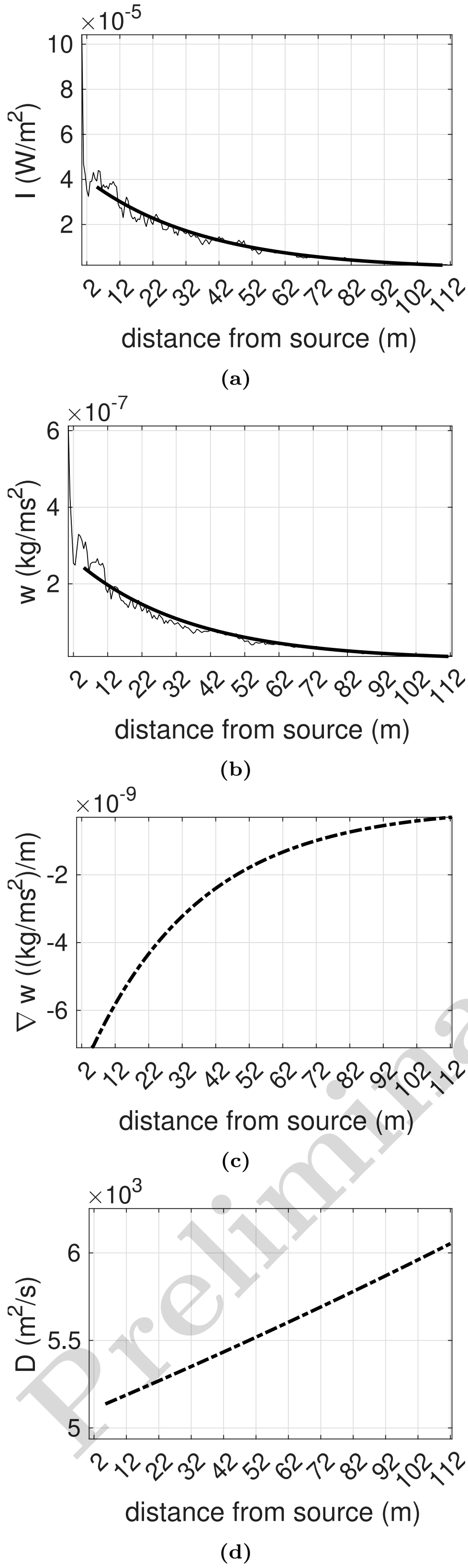

Figure 4: (a) Intensity, (b) energy density, (c) gradient of energy density, and (d) diffusion coefficient of the tunnel 112.5 m x 2.5 m x 2-3 m for 1000 Hz; ——Measured values, ▬▬ Fitted curve, ▪▪▪ Calculated from fitted curves.

5. Validation of diffusion coefficient in diffusion equation

The validation study consists of comparing the numerical estimates of SPL and T_{20} obtained from the diffusion equation model using the measured diffusion coefficient with the corresponding experimental measurement results. These are also compared to the results of the diffusion equation model using a theoretical diffusion coefficient. To perform this validation, the tunnel is numerically simulated using the finite volume method implementation of the diffusion equation model from the 'acousticDE' GitHub repository [14] developed by the Building Acoustics Group at the Eindhoven University of Technology. The geometry of the simulated tunnel has the same cross-section as the real tunnel, including the curved ceiling and a length of 116.5 m. The total length of 116.5 m includes 2 m before the source position, 112.5 m of the measurement region, and an additional 2 m after that. The resulting 3D model has a volume of 802.42 m^3 and a total surface area of 1177.05 m^2. With these dimensions, the value of the theoretical diffusion coefficient calculated by Equation (2) is 331.76 m^2/s. The choice of truncating the tunnel length to 116.5 m did not influence the value of the theoretical diffusion coefficient, since the ratio between the volume and the total surface area remains constant independently of the length of the tunnel. To simplify the model, the recesses for the passage of cables along the lateral surfaces have not been considered. The diffusion equation was solved using a tetrahedral mesh with a mesh length of 0.5 m to discretise the volume. For time discretisation, a Δt of 1/20000 s was used for a total simulation duration of around 7 seconds (corresponding to the higher value of T_{20} measured in the tunnel). In addition to geometry, the diffusion equation requires the diffusion coefficient and the absorption coefficients as input. The diffusion coefficients estimated from the intensity and energy density measurements, as explained in Section 4 and shown in Figure 4 (d) are taken. From the beginning of the tunnel to the starting point of the fitting, the value of the diffusion coefficient in the simulation is considered equal to the value at the starting point of the fitting; the same applies to the last value of the diffusion coefficient after the last point of fitting. The absorption coefficients of the tunnel surfaces could not be measured during the campaign. Therefore, the absorption coefficients for each frequency are estimated from the average values of T_{20} over the distance from the source and calculated using the Sabine formula. From the reverberation time measurements of the long tunnels, assuming that since the extensive tunnel network, any return of sound to the measurement zone is negligible and therefore imposing the absorption coefficient of the extremities of the tunnel equal to 1, the absorption coefficient of the longitudinal surfaces of the tunnel was estimated to be 0.0226 at 1000 Hz. Since Sabine assumes a diffuse field, this estimate might not be accurate. Therefore, a physically realistic range of absorption coefficients for concrete materials was taken from the literature [15], where typical values range between 0.01 and 0.05. Therefore, two additional simulations were performed, corresponding to the absorption values of 0.01 and 0.05 at 1000 Hz. The comparison between simulations and measurement results is shown in Figure 5 in terms of SPL and T_{20}. This figure shows the influence of both absorption and diffusion coefficient on the simulated results.

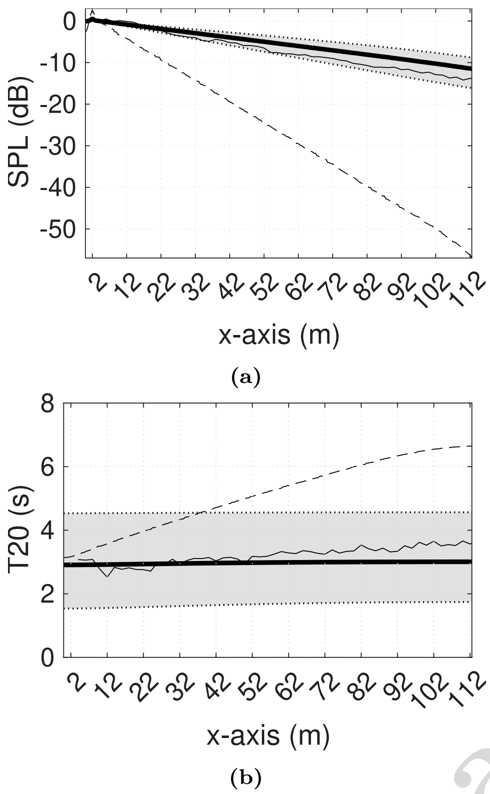

Figure 5: (a) SPL comparison, (b) T_{20} comparison at 1000 Hz between: ——Measurements, ▬Diffusion Model with measured fitted diffusion coefficient ($\bar{\alpha} = 0.0226$), ⋯⋯Diffusion Model with measured fitted diffusion coefficient ($\bar{\alpha} = 0.01$) and ($\bar{\alpha} = 0.05$), - - -Diffusion Model with theoretical diffusion coefficient. The range between the absorption coefficients 0.01 and 0.05 is shown as a gray area.

Figure 5 shows that the absorption coefficient of 0.0226, as estimated from Sabine's formula, gives a good estimate for T_{20}. Good agreement is observed near the source, while discrepancies increase farther away from the source for a maximum of 0.6 s. For the SPL, the simulated values with 0.0226 absorption coefficient show good agreement with the values of the measured results up to 50 m from the source. The observed difference in SPL between the simulated and measured results increases with the distance from the source to a maximum value of 2.3 dB at 112.5 m from the source. The difference in SPL and T_{20} can be attributed to the fact that the absorption coefficient was estimated rather than measured directly. At higher frequencies (not shown in these figures), the SPL discrepancy decreases slightly, ranging from approximately 0.6 to 1 dB instead of exactly 2.3 dB. Figure 5 also shows the results of the diffusion equation when using the theoretical diffusion coefficient value of 311.76 m^2/s; in this case, the difference between the simulated and measured results is very large, reaching 50 dB for SPL and more than 3 s for T_{20}. This confirms that the theoretical diffusion coefficient cannot be used for elongated rooms and that the real diffusion coefficient changes over distance from the source because of the physical characteristics of the room. The measured diffusion coefficient leads to more realistic and accurate results of the SPL and T_{20} in the tunnel.

6. Further work and discussion

As shown in Figure 4, the measured values of the diffusion coefficient are consistently higher than the theoretical diffusion coefficient of 311.76 m^2 / s, which contrasts with most findings in the literature. At 2-3 m distance from the source, the coefficient is approximately 5000 m^2/s, corresponding to a value of $16 \cdot D_{th}$ and increases to roughly 6000 m^2/s ($19 \cdot D_{th}$) at a distance of 112 m. A possible explanation for this offset is surface scattering. In the diffusion equation model, the reflections are assumed to be fully scattered (scattering coefficient equal to 1), while, in practice, the boundary surfaces are relatively smooth. In fact, looking at the scattering elements in the tunnel in Figure 2 there is only a significant scattering effect in the vertical plane. Considering that the dominant propagation direction is the length of the tunnel, according to the literature [16], [17], a scattering coefficient for the smooth concrete surfaces of the tunnel of 0.05 is considered to be realistic. The effect of scattering on the diffusion coefficient was investigated by Foy [18], who proposed to incorporate scattering by multiplying the theoretical diffusion coefficient by a factor of $1/(\alpha + (1-\alpha)s)$. This modification was determined analytically for an unbounded medium and very low absorption coefficients. Using a scattering coefficient of 0.05 and an average value of the absorption coefficient at 1000 Hz of 0.0226, derived from the measurements of reverberation time, this factor is evaluated to approximately 15. Therefore, the high diffusion coefficient around $16 \cdot D_{th}$ could be attributed to surface scattering.

7. Conclusions

For elongated rooms, the diffusion equation model, which uses the theoretical diffusion coefficient, incorrectly estimates its variable properties SPL and T_{20}. Earlier, researchers have numerically estimated that the diffusion coefficient in elongated tunnels should be spatially dependent. In this work, experimental measurements are made in a 300 m long tunnel of the Schoenenbourg fort of the Maginot line. In particular, intensity and energy density were measured over the length of the tunnel to calculate the diffusion coefficient, while SPL and T_{20} were measured to have reference result values. Through these experimental measurements, it was demonstrated that the diffusion coefficient is not constant and that it changes over the distance from the source. In addition, the measured diffusion coefficient values are used in the diffusion equation to verify that the SPL and T_{20} are the same compared to the measured results. SPL and T_{20} on the length of the tunnel are successfully reproduced with much higher accuracy compared to when using the theoretical diffusion coefficient.

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