EXPERIMENTAL DETERMINATION OF DYNAMIC VISCOUS RESISTANCE AND THERMAL RELAXATION CONDUCTANCE IN POROUS MATERIALS

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Abstract

Porous materials are widely used in acoustic and thermoacoustic applications due to their ability to dissipate acoustic energy through viscous and thermal interactions between the fluid and the solid skeleton. These mechanisms are commonly described by visco-thermal parameters such as the viscous resistance and the thermal relaxation conductance, which play a central role in equivalent fluid models of porous media. However, their direct experimental determination remains challenging.

This work presents two experimental methodologies enabling the estimation in the low frequency regime of viscous resistance and thermal relaxation conductance in porous materials subjected to oscillating flow. The techniques rely on acoustic pressure measurements performed in a modified two-microphones technique in a standing wave tube. Two different measurement configurations allow the viscous and thermal contributions to be independently evaluated.

The proposed approach is tested experimentally on a polyester fibers sample. The results show good agreement between experimental estimations and theoretical predictions, demonstrating that the proposed methods allow direct estimation of visco-thermal parameters of porous materials. The presented techniques provide a useful experimental tool for the characterization of porous materials in acoustic and thermoacoustic applications.

Keywords: *porous materials, viscous resistance, thermal relaxation conductance, oscillating flow, acoustic measurements*

1. Introduction

Porous materials are widely employed in many engineering fields thanks to their low density, large surface area and permeability. They play an important role in acoustic absorption, thermal management and thermoacoustic energy conversion.

When an oscillating flow propagates inside a porous structure, viscous stresses and thermal exchanges occur between the fluid and the solid skeleton. These mechanisms lead to acoustic energy dissipation and are commonly described through visco-thermal parameters used in equivalent fluid models of porous media.

Among these parameters, viscous resistance and thermal relaxation conductance govern the interaction between the acoustic field and the microstructure of the material. In classical acoustic measurements, such as impedance tube techniques, the global acoustic behaviour of the material can be determined, but the direct estimation of the individual viscous and thermal contributions remains difficult [1], [2].

The objective of this work is to present two experimental methods allowing the estimation of viscous resistance and thermal relaxation parameters in porous materials subjected to oscillating flow. The techniques rely on pressure measurements performed through a modified two-microphones technique. The approach is validated experimentally on different porous materials.

2. Theoretical background

An infinitesimal slice dx of a porous material subjected to an acoustic wave can be represented, through the electroacoustic analogy, as a two-port element (double bipole) characterized by a longitudinal impedance associated with viscous effects and a transversal admittance associated with thermal exchanges [3], see Fig. 1.

When a plane acoustic wave propagates through a porous material, viscous and thermal interactions between the fluid and the solid skeleton lead to acoustic energy dissipation.

The variation of acoustic power along the material can be expressed as

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$$\frac{d\dot{E}}{dx} = \frac{1}{2}\Re\left[\tilde{U}\frac{dp}{dx} + \tilde{p}\frac{dU}{dx}\right], \tag{1}$$

where p is the acoustic pressure, U is the acoustic volume velocity, and $\sim$ indicates the complex conjugate. Following classical thermoacoustic formulations, the dissipated power can be decomposed into viscous and thermal contributions

$$\frac{d\dot{E}}{dx} = -\frac{1}{2}\left(r_\nu|U|^2 + \frac{1}{r_\kappa}|p|^2\right), \tag{2}$$

where r_ν represents the viscous resistance and $1/r_\kappa$ the thermal relaxation conductance. These relations highlight that viscous dissipation depends on the squared acoustic volume velocity while thermal dissipation depends on the squared acoustic pressure.
Comparing Eqs. (1) and (2) allows the viscous and thermal contributions to be identified separately. In particular, the viscous dissipation is associated with the interaction between the acoustic volume velocity and the pressure gradient, whereas the thermal dissipation is related to the interaction between the acoustic pressure and the volume velocity gradient. This leads to

$$r_\nu|U|^2 = \Re\Big[\tilde{U}\frac{dp}{dx}\Big] \quad \Rightarrow \quad r_\nu = \Re\Big[\frac{dp/dx}{U}\Big], \tag{3}$$
$$\frac{1}{r_\kappa}|p|^2 = \Re\Big[\tilde{p}\frac{dU}{dx}\Big] \quad \Rightarrow \quad \frac{1}{r_\kappa} = \Re\Big[\frac{dU/dx}{p}\Big]. \tag{4}$$

These relations highlight the physical meaning of the visco-thermal parameters. The viscous resistance r_ν is directly related to the ratio between the pressure gradient and the acoustic volume velocity, which reflects the friction effects experienced by the oscillating fluid inside the pores. In other words, r_ν quantifies the opposition of the porous structure to the oscillatory flow.
Conversely, the thermal relaxation conductance $1/r_\kappa$ depends on the ratio between the gradient of volume velocity and the acoustic pressure. This term describes the thermal exchanges between the fluid and the solid skeleton associated with compressions and expansions of the oscillating fluid inside the pores.
Therefore, viscous dissipation is primarily governed by the pressure gradient imposed along the material, while thermal dissipation is associated with spatial variations of the acoustic volume velocity.
From a modeling point of view, these quantities can be accurately predicted using the following relations [3], [4]

$$r_\nu = -\omega\Im[\rho], \tag{5}$$
$$\frac{1}{r_\kappa} = \omega\frac{\Im[K]}{|K|^2}, \tag{6}$$

where ρ and K are the complex density and bulk modulus which can be obtained using the semi-phenomenological Johnson–Champoux–Allard–Lafarge (JCAL) model [5], [6], [7].

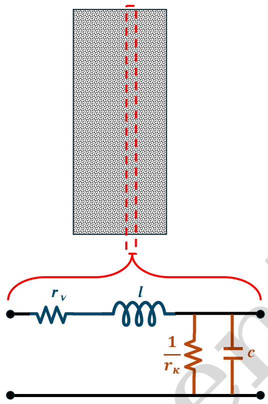

Figure 1: Electroacoustic equivalent circuit of an infinitesimal slice of porous material, modeled as a two-port element (double bipole) with a longitudinal impedance $r_\nu + i\omega l$ associated and a transversal admittance associated $1/r_\kappa + i\omega c$. Viscous and thermal dissipations are related respectively to the real parts r_ν and $1/r_\kappa$.

3. Measurement method

The estimation of the visco-thermal parameters is based on the relations derived in the previous section, which link the viscous resistance r_ν and the thermal relaxation conductance $1/r_\kappa$ to the spatial gradients of pressure and volume velocity.

3.1. Thermal relaxation measurement

The estimation of the thermal relaxation conductance is performed using the classical two-microphone impedance tube configuration, where the sample is backed by a rigid termination, as in Fig. 2.a. In this case the acoustic volume velocity at the rear surface of the sample vanishes ($U = 0$) [8].
At sufficiently low frequencies, the acoustic pressure can be considered approximately uniform inside the porous material. Under this assumption, the relation obtained in Eq. (4) can be rewritten as

$$\frac{1}{r_\kappa} = \Re\left[\frac{d}{dx}\left(\frac{U}{p}\right)\right]. \tag{7}$$

Since the specific acoustic impedance is defined as $Z_s = p/U$, the previous relation becomes

$$\frac{1}{r_\kappa} = \frac{d}{dx}\Re\left[\frac{1}{Z_s}\right]. \tag{8}$$

Assuming that the pressure remains approximately constant inside the material, the spatial variation of the volume velocity can be considered linear. The thermal relaxation conductance can therefore be approximated as

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$$\frac{1}{r_\kappa} = \frac{\Re[1/Z_s]_{x=0} - \Re[1/Z_s]_{x=L}}{L}. \tag{9}$$

At the rigid termination ($x = L$, where L is the sample thickness) the volume velocity vanishes, which implies $\Re[1/Z_s]_{x=L} \to 0$. Consequently

$$\frac{1}{r_\kappa} = \frac{\Re[1/Z_s]_{x=0}}{L}, \tag{10}$$

where Z_s is the surface impedance measured at the front face of the sample using the two-microphone impedance tube method.

3.2. Viscous resistance measurement

The viscous resistance can be estimated using a complementary configuration in which the rear side of the porous material is left open, as in Fig. 2.b. Under this boundary condition the acoustic pressure at the rear surface tends to vanish ($p = 0$).

At low frequencies the acoustic volume velocity can be considered approximately constant throughout the material, while the pressure varies along the sample thickness. The viscous resistance can therefore be expressed as

$$r_\nu = \Re\left[\frac{d}{dx}\left(\frac{p}{U}\right)\right]. \tag{11}$$

Using the definition of acoustic impedance, this relation can be rewritten as

$$r_\nu = \frac{d}{dx}\Re[Z_s]. \tag{12}$$

Assuming a linear variation of pressure along the material thickness, the viscous resistance becomes

$$r_\nu = \frac{\Re[Z_s]_{x=0} - \Re[Z_s]_{x=L}}{L}. \tag{13}$$

Since the pressure vanishes at the open termination ($x = L$), it follows that $\Re[Z_s]_{x=L} \to 0$, leading to the final relation

$$r_\nu = \frac{\Re[Z_s]_{x=0}}{L}, \tag{14}$$

where Z_s is again obtained from the two-microphone impedance tube measurements.

These two complementary configurations therefore allow the direct estimation of the viscous resistance and the thermal relaxation conductance of porous materials.

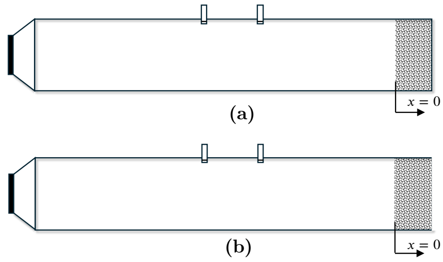

Figure 2: Schematic representation of the two-microphone impedance tube configurations used for the estimation of visco-thermal parameters: (a) rigidly backed sample for the measurement of thermal relaxation conductance $1/r_\kappa$, and (b) open termination for the measurement of viscous resistance r_ν.

4. Experimental results and discussion

Preliminary experimental measurements are carried out in order to validate the proposed methodology for the estimation of the visco-thermal parameters.
The tests are performed on a polyester fiber sample with thickness $L = 40\,\mathrm{mm}$ and diameter 100 mm, placed inside a standing wave tube of the same diameter. Measurements are conducted under ambient conditions using air as working fluid, restricting the analysis to the low-frequency range in order to satisfy the assumptions used in the derivation of Eqs. (14) and (10).
The investigated material had been previously characterized [9] and its acoustic behaviour can be accurately described using Eqs. (5), (6) where the input transport parameters used in the JCAL model for this material are reported in Table 1. The experimentally estimated visco-thermal parameters were therefore compared with the theoretical predictions obtained from this model.

Table 1: Transport parameters of the investigated polyester fibrous material used in the JCAL model.

Parameter	Value
Porosity φ	0.99
Tortuosity α_∞	1.02
Airflow resistivity σ	$1200\,\mathrm{Pa\cdot s/m^2}$
Static thermal permeability k_0'	$1.51\times10^{-8}\,\mathrm{m^2}$
Viscous characteristic length Λ	0.2 mm
Thermal characteristic length Λ'	0.4 mm

Figures 3a and 3b present the comparison between the experimental measurements and the theoretical predictions in log–log scale.
Figure 3a shows the behaviour of the viscous resistance r_ν as a function of frequency. In the low-frequency limit, r_ν tends to a constant value, which corresponds to the airflow resistivity of the material. This behaviour is in agreement with the experimental measurements, confirming the ability of the proposed method to correctly estimate the viscous parameter in the quasi-static regime.
Figure 3b presents the thermal relaxation conductance

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$1/r_\kappa$ as a function of frequency. In logarithmic scale, the experimental results exhibit an approximately linear trend. This behaviour is consistent with the theoretical predictions of the JCAL model, in which the slope of the curve is directly related to the static thermal permeability of the material. The agreement between experimental and theoretical trends further supports the validity of the proposed approach for the estimation of thermal effects.
Overall, the observed behaviours of both r_ν and $1/r_\kappa$ confirm that the proposed measurement techniques are able to capture the fundamental visco-thermal mechanisms governing acoustic dissipation in porous materials.

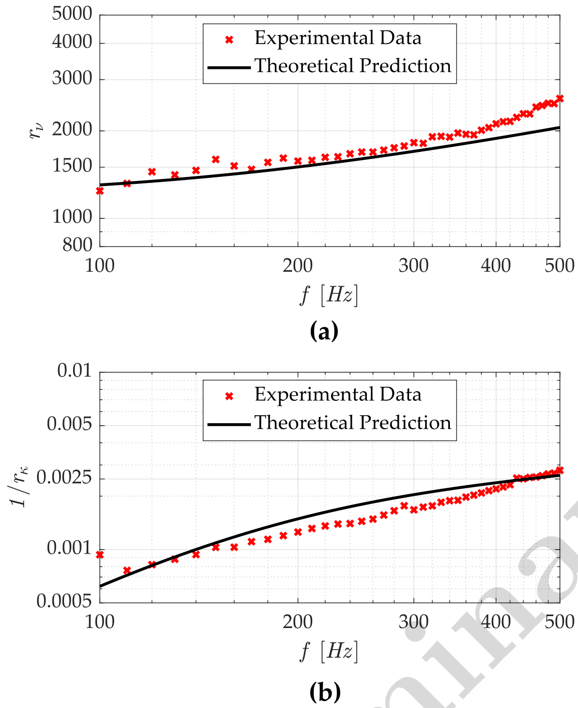

Figure 3: Comparison between experimental measurements and JCAL model predictions for the investigated polyester fibrous material. (a) Viscous resistance r_ν and (b) thermal relaxation conductance $1/r_\kappa$ as functions of frequency. The solid black line represents the theoretical prediction, while red circular markers denote the experimental data.

5. Conclusion

This work presented two experimental techniques enabling the estimation of viscous resistance and thermal relaxation parameters in porous materials subjected to oscillating flow. The methods are based on acoustic pressure measurements performed in a modified standing wave tube and rely on the lumped element behaviour of the material at low frequencies. Experimental measurements carried out on a polyester fiber sample show good agreement between experimental estimations and theoretical predictions. The proposed approach provides a useful experimental tool for the characterization of porous materials and for the direct determination of visco-thermal parameters relevant to acoustic and thermoacoustic applications.Future developments will involve testing this measurement method on different types of specimens and systematically analysing the frequency range within which the measurement is valid

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