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A 2D NUMERICAL STUDY OF A DIFFUSE FIELD INDEX

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Abstract

Absorption coefficient measurements of material samples in reverberation chambers depend on diffuse field theory. The diffuseness of a sound field depends on the geometry and sound absorption of a given chamber. In this numerical study, a spatially averaged intensity-based diffuse field index is computed as a function of random-incidence absorption coefficient and frequency in a 2D geometry, specifically the Bunimovich stadium. It is shown that the diffuseness of the sound field is reduced when the surface absorption is non-uniform. The effect of scattering and the number of sources on the diffuse field index are also briefly considered. These results highlight the challenge practitioners face when attempting to reliably measure absorption coefficients in reverberation chambers using methods that depend on diffuse field theory.

Keywords: *diffuse field, Bunimovich stadium, non-uniform absorption*

1. Introduction

Diffuse sound field theory is important in many branches of acoustics. It plays a crucial role in measuring absorption coefficients, sound power, and sound transmission [1], as well as in the characterization and equalization of sound transducers [2], [3]. Evaluating and improving the diffuseness of a sound field are tasks that have kept the community occupied for many years [4], and numerous diffuse field definitions [5], [6], [7] and indices [8] can be found in the literature. A sound field is considered diffuse when reflections arrive from all directions with equal intensity. While a perfectly diffuse sound field is difficult to realize in practice [9], diffuse field theory provides valuable insights into sound propagation in enclosed spaces, especially in irregularly shaped rooms with uniform absorption and high modal density (i.e., at high frequencies) [10]. Sound fields generated in reverberation chambers are approximately diffuse [11].

The diffuseness of a reverberation chamber absorption coefficient measurement is the (indirect) subject of this study. We quantify the diffuseness of a sound field using the intensity-based Diffuse Field Index (DFI), which is presented in Sec. 2. The finite element method is used to predict the sound field quantities needed to compute the DFI for a range of absorption coefficients and frequencies. However, due to the prohibitive computational costs of high-frequency solutions in 3D models, a 2D model is considered. We consider the Bunimovich stadium [12], presented in Sec. 3, which has the necessary properties to support a diffuse field, i.e., ergodicity and mixing [13], [14].[1] We mimic the reverberation chamber absorption coefficient measurement by computing the DFI for uniform and non-uniform surface absorption in Sec. 4. A brief study of the effect of scattering and multiple sources is provided in Sec. 5, and this study is concluded in Sec. 6.

2. Theory

We begin by introducing the field index proposed by Vigran [16]

$$\mathrm{F} = \frac{\|\mathbf{I}\|_2}{\mathrm{E}c}, \tag{1}$$

where $\mathbf{I}$ is the active intensity, E is the energy density, c is the speed of sound, and $\|\cdot\|_2$ is the Euclidean norm. The active intensity vector at a given position, $\mathbf{x}$, and frequency, ω, is given by

$$\mathbf{I}(\mathbf{x},\omega) = \frac{1}{2}\mathrm{Re}\left(P(\mathbf{x},\omega)\mathbf{V}^*(\mathbf{x},\omega)\right) \ , \tag{2}$$

where P is the complex-valued acoustic pressure, $\mathbf{V}$ is the complex-valued particle velocity vector, and $(\cdot)^*$ denotes the complex conjugate. The energy density is

[1] These concepts form the basis of Badeau's recently proposed statistical wave field theory [15] (we note, however, that these assumptions are relaxed in advanced versions of the theory).

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given by

$$\mathrm{E}(\mathbf{x},\omega)=\frac{|P(\mathbf{x},\omega)|^2}{4\rho c^2}+\frac{\rho\|\mathbf{V}(\mathbf{x},\omega)\|_2^2}{4}\ , \qquad (3)$$

with density ρ. In this work, we choose to use a spatially averaged, intensity-based DFI, given by

$$\mathrm{DFI}=1-\langle \mathrm{F}\rangle\ , \qquad (4)$$

where $\langle\cdot\rangle$ is a spatial average. Additionally, we consider the ratio of the potential energy (first term on the right-hand side of Eq. (3)) to the kinetic energy (second right-hand side term of Eq. (3)), written as

$$\mathrm{R}_{\mathrm{pk}}=\frac{\mathrm{E}_{\mathrm{p}}}{\mathrm{E}_{\mathrm{k}}}. \qquad (5)$$

In a diffuse field, we expect R_{pk} to be close to unity, which represents a balance between the potential and kinetic energy.
To obtain the pressure and velocity data needed to compute Eq. (4) and Eq. (5), the finite element method is used. Triangular elements with fifth-order shape functions are used. Ten degrees of freedom per wavelength are used to ensure minimal dispersion error and high-quality estimates of the pressure gradient and, therefore, the velocity.

3. Bunimovich stadium

Since geometry has a significant effect on the diffuseness of an interior sound field, we choose to use a geometry that provides the necessary properties for a diffuse field, which are ergodicity and a mixing condition. Ergodicity, discussed by Joyce [17], allows sound particles to explore every point and direction in a space, while mixing refers to how closely traveling particles disperse to create a uniform distribution. The Bunimovich stadium under test is shown in Fig. 1.
Ergodicity is demonstrated in Fig. 2a. A particle of sound is emitted from source position S_0 at an angle of $\pi/5$ (clockwise from the y-axis). A ray tracing algorithm is employed to trace the path of 100 (specular) reflections. We observe that the particle has explored many positions in the stadium. We expect that as the number of reflections goes to infinity, the particle will visit every possible position of the stadium, and will approach every position from every possible angle.
The mixing property is illustrated in Fig. 2b. The same source position is considered, but two particles are emitted with a very small difference between their angles of propagation, specifically $\pi/5$ and $\pi/5.1$. Although these two particles start off close to one another, after five reflections their propagation paths diverge significantly.
Let us now consider a finite element solution to this problem. Let the walls of the stadium be rigid, and the source be a monopole oscillating at 5 kHz. In this case, the DFI is unity. In fact, this holds for all rigid-walled geometries, since the intensity is zero. However, the Sound Pressure Level (SPL) in the stadium can

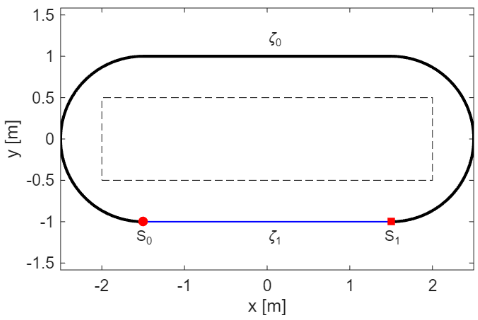
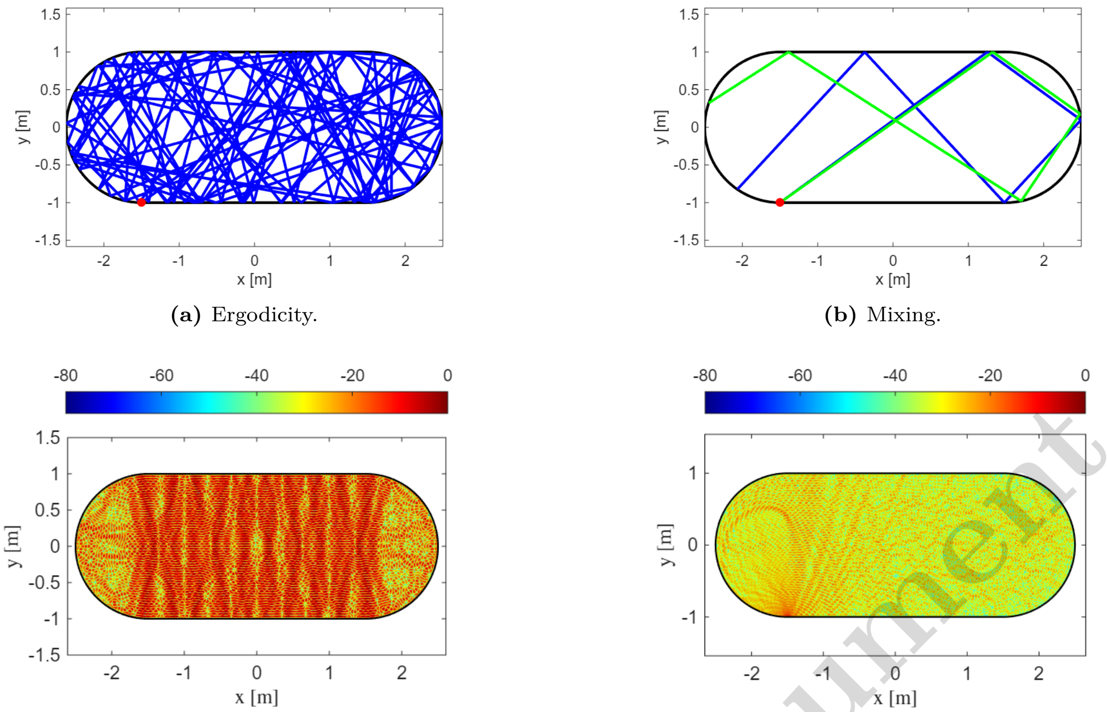


Figure 1: Bunimovich stadium. The boundary is divided into two parts. Impedance ζ_0 is specified on the upper part, and ζ_1 is specified on the lower part. Two source positions are considered, these are labeled S_0 and S_1. The dashed line delineates the region in which the DFI is computed.

be computed. This is shown in Fig. 2c. We observe spatial variations in the SPL.
Introducing absorption to the walls of the stadium will change the diffuseness of the sound field. A simple demonstration is provided here. We specify a normalized impedance of $\zeta_0=\zeta_1=10$ on all boundaries, i.e., we apply a uniform impedance. This setup results in a flow of energy from the source to the walls, causing the intensity to be non-zero. The SPL is illustrated in Fig. 2d. We observe that the features of the sound field are noticeably different from those of the sound field in the presence of rigid walls.

4. DFI study

To study the DFI of the sound field in the stadium, we vary the boundary surface impedance, from $\zeta_0=\zeta_1=1.57$ to $\zeta_0=\zeta_1=7.98\times10^3$, and compute the DFI across a range of third-octave band center frequencies, from 20 Hz to 5 kHz (no averaging within bands is used, only predictions at single frequencies are reported). Since we are using the finite element method, we can calculate the DFI over the entire domain. However, there are certain regions we choose to avoid, as including them could complicate the interpretation of the results. Thus, we omit the boundaries and the source position from the DFI computation (cf. Fig. 1). The DFI data is presented in Fig. 3 as a function of frequency and random incidence absorption coefficient (which is computed from the impedance using the Paris formula [18]).
For a uniform impedance, the predicted DFI is presented in Fig. 3a. We remind the reader that each point on this plot corresponds to a spatial average over the rectangular observer region. At high frequencies (above approximately 1 kHz), the sound field is diffuse for lightly damped surfaces, i.e., for small absorption coefficients. As the absorption of the surfaces increases, the sound field becomes less diffuse. A DFI of approximately 0.7 is found away from the two extremes of the absorption coefficient. The plot also includes dashed lines representing the first 20 resonance frequencies of the rigid-walled stadium. Note, the DFI



(a) Ergodicity.

(b) Mixing.

(c) SPL - rigid wall.

(d) SPL - impedance wall.

Figure 2: The Bunimovich stadium: ray tracing and wave solutions. (a) A single ray is emitted from S_0 (shown in red). The first 100 reflections are shown. (b) Two rays are emitted from the source. The first five reflections are shown. (c) The SPL at 5 kHz in a rigid-walled stadium. (d) The SPL at 5 kHz in a stadium with absorbing walls ($\zeta_0 = \zeta_1 = 10$).

does not appear to align with the resonance frequencies. At low frequencies, the sound field appears to be diffuse even for moderately damped surfaces. The cause of this is the stadium's modes; this is confirmed upon consultation of R_{pk} in Fig. 3b. At low frequencies R_{pk} deviates from unity, and indicates that the kinetic energy is greater than the potential energy. At low frequencies, what appears to be a diffuse field is actually due to the modal field. We see that at high frequencies for low absorption coefficients, $R_{pk} \approx 1$, as expected.

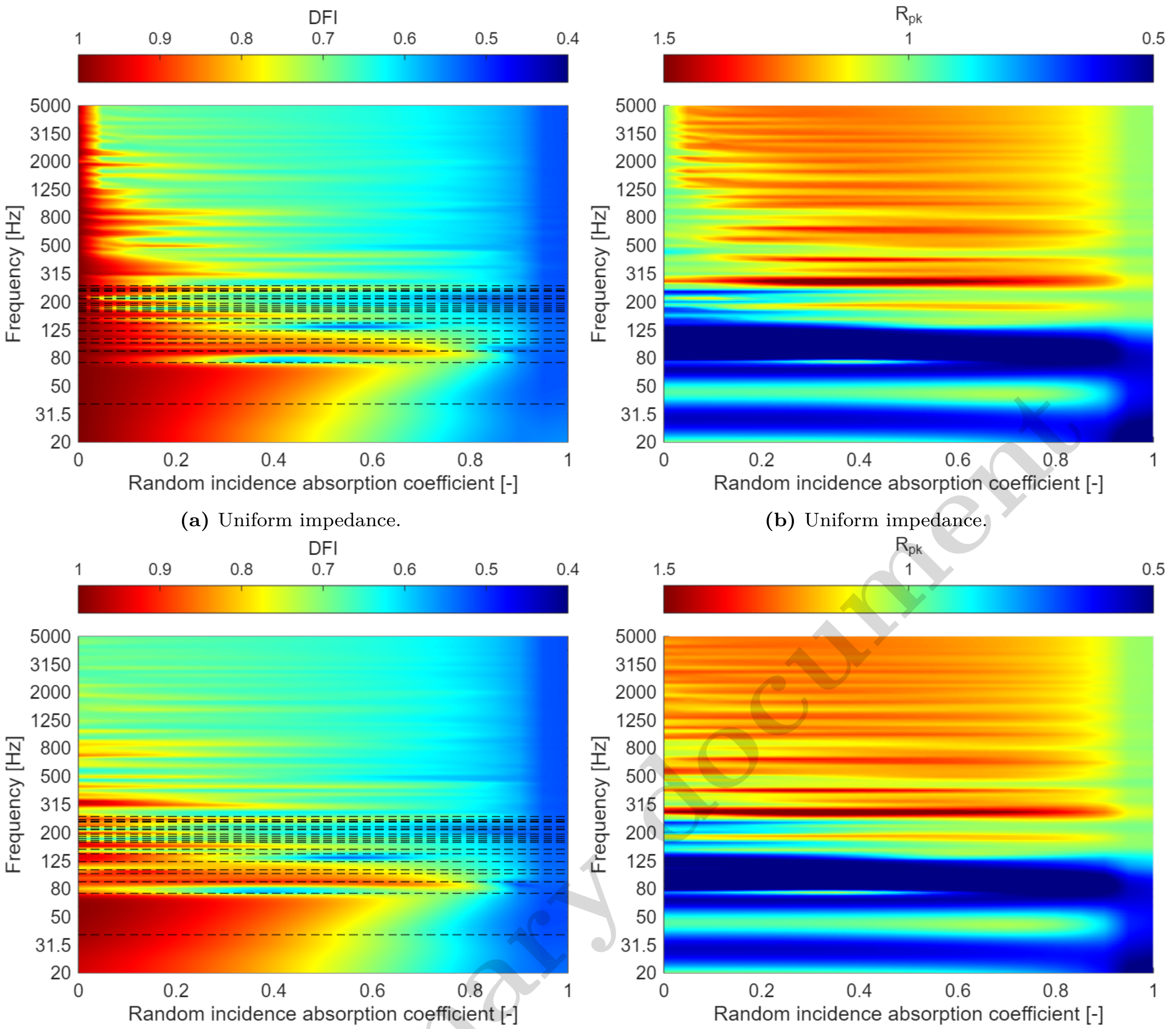
Let us now consider the case for which non-uniform impedance is specified. Let $\zeta_1 = 10$ while we vary ζ_0 as before. The DFI computed in this case is shown in Fig. 3c. The diffuse field observed at high frequencies in the uniform impedance case has disappeared in the non-uniform impedance case. This is confirmed by the R_{pk} shown in Fig. 3d, for which the potential energy is greater than the kinetic energy. At low frequencies, the DFI and R_{pk} are similar to the uniform impedance case, and do not appear to depend on the uniformity of the impedance. These results show that the diffuse field index is considerably lower in a room with non-uniform impedance. This finding is concerning, as it implies that placing an absorbing sample in a room alters the sound field's diffuseness. Such changes could impact absorption coefficient measurements in a reverberation chamber, which make use of Sabine's formula and thereby require a diffuse field.

5. Scattering and sources

Two attempts to make the sound field more diffuse are considered in this section: distorting the boundary surface, and adding a second source, S_1.

The motivation for distorting the boundary is to introduce scattering into the problem, which is inspired by Ref. [19]. To achieve this, the boundary nodes have been randomly shifted. The distorted geometry is then remeshed and used to predict the pressure and velocity generated by the point source. For a uniform impedance case ($\zeta_0 = \zeta_1 = 300$), the SPL is presented in Fig. 4a. It can be see that there is a region in the center of the domain where the pressure is lower than at the boundaries. If we set $\zeta_1 = 10$, we find that the pressure distribution is less uniform, as expected. There seems to be more energy distributed on the left side of the room, closer to the source. A logical step would be to add another sound source, as suggested in the standard for reverberation chamber measurements, which advocates for multiple sources. Point source S_1 is excited with the same volume velocity as S_0. The SPLs from both the uniform and non-uniform impedances are presented in Fig. 4c and Fig. 4d. As in the single-source case, adding a second source does not make the pressure levels more uniform. Additionally, data not presented here indicate that the added scattering does not make the sound field more diffuse.

Abandoning the scattering, we return to the smooth surface of the original stadium, but we keep the ad-



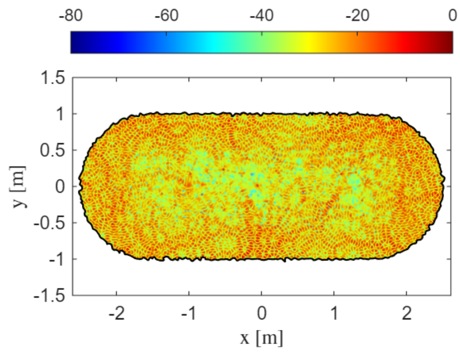
(a) Uniform impedance. **(b)** Uniform impedance. **(c)** Non-uniform impedance. **(d)** Non-uniform impedance.

Figure 3: Diffuse field metrics of the Bunimovich stadium as a function of random incidence absorption coefficient and frequency. (a) DFI for uniform impedance. (b) R_{pk} for uniform impedance. (c) DFI for non-uniform impedance. (d) R_{pk} for non-uniform impedance.

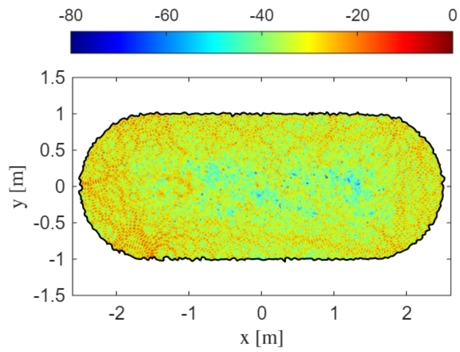
ditional source. The SPLs in this case are shown in Fig. 4e and Fig. 4f for the uniform and non-uniform impedance cases, respectively. In both cases, the variation in SPLs is much more uniform, possibly providing a closer approximation to a diffuse field. Finally, we present DFI and R_{pk} for the smooth boundary with two sources in Fig. 5. We see that the introduction of a second source does make the uniform impedance sound field more diffuse than the single source case. However, for the non-uniform impedance, the sound field is only (approximately) 70% diffuse.

6. Conclusion

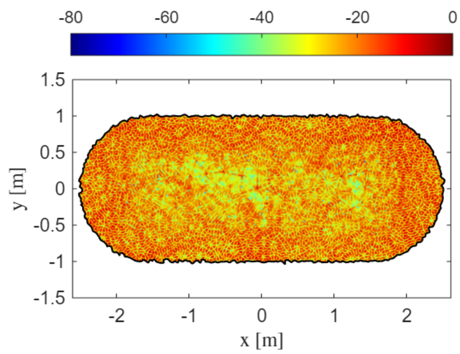
An intensity-based diffuse field index and the potential-to-kinetic energy ratio have been used to quantify the diffuseness of a sound field in a Bunimovich stadium, subject to varying surface absorption conditions. It has been shown that surface absorption affects the diffuseness of a sound field. An approximately diffuse field can be obtained when the absorption is small and uniform. However, when the absorption is non-uniform, the sound field becomes less diffuse. Additionally, while adding more sources increases diffuseness in the uniform absorption case, it does not appear to increase diffuseness in the non-uniform case. This has implications for reverberation chamber absorption coefficient measurements that make use of Sabine's formula, and thus rely on diffuse field theory. The findings of this study suggest that simply placing an absorbing material into a reverberation chamber changes the diffuseness of the sound field, thereby making the diffuse field assumption less applicable.



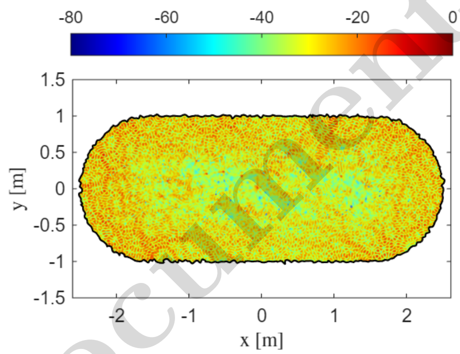
(a) Scattering, uniform impedance, 1 source.



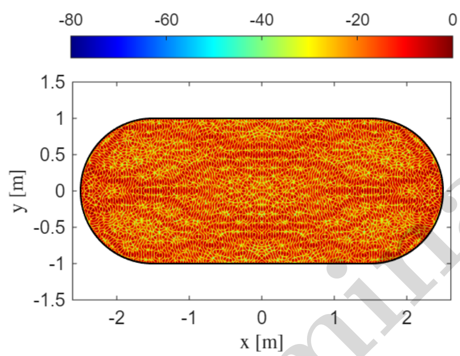
(b) Scattering, non-uniform impedance, 1 source.



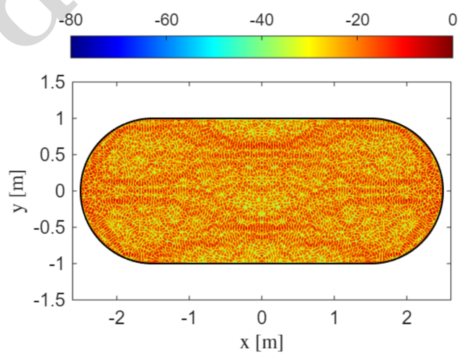
(c) Scattering, uniform impedance, 2 sources.



(d) Scattering, non-uniform impedance, 2 sources.



(e) Uniform impedance, 2 sources.



(f) Non-uniform impedance, 2 sources.

Figure 4: SPLs of the stadium with and without scattering, and with one or two sources.

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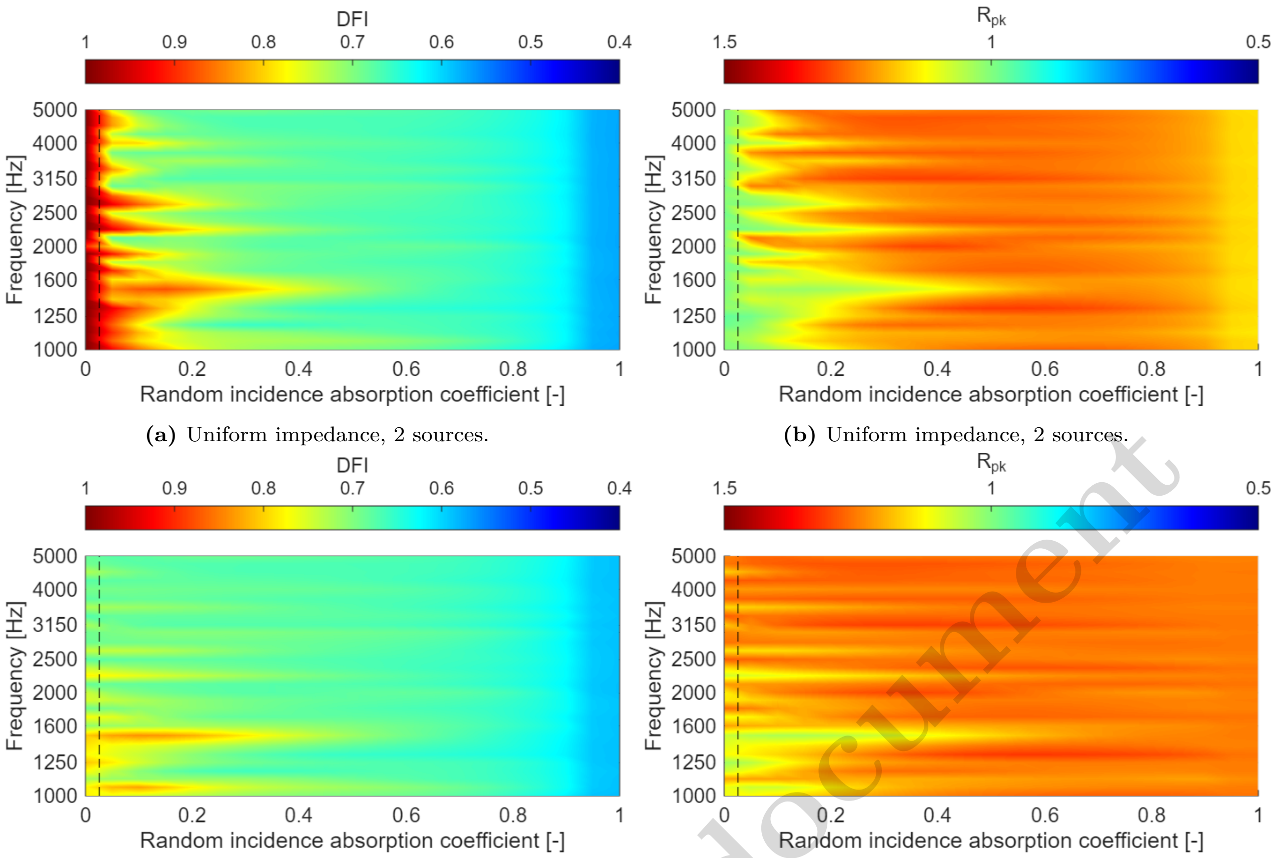

(a) Uniform impedance, 2 sources.

(b) Uniform impedance, 2 sources.

(c) Non-uniform impedance, 2 sources.

(d) Non-uniform impedance, 2 sources.

Figure 5: Diffuse field metrics for the stadium with two sources. The dashed vertical line indicates the absorption coefficient given by $\zeta_0 = 300$. (a) DFI for uniform impedance. (b) R_{pk} for uniform impedance. (c) DFI for non-uniform impedance. (d) R_{pk} for non-uniform impedance.

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