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RAY TRACING OF ACOUSTIC WAVES GUIDED IN A JET

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Abstract

Ray tracing is performed from a point source located on the axis of a cylindrical shear layer at a Mach number ranging up to $M_j = 2$. In all cases, some of the rays originally directed upstream are guided by the jet flow. These guided jet rays (GJR) can be entirely or partially confined inside the jet depending on the Mach number and the ray initial angle. They can travel in the upstream or downstream directions, or not propagate axially. For subsonic Mach numbers, upstream-travelling GJR are found in all cases, and downstream-travelling GJR are observed above a threshold Mach number. For supersonic Mach numbers, both upstream- and downstream-travelling GJR exist. These results are consistent with the properties of the guided jet waves described in previous studies.

Keywords: *jet, guided jet waves, sound propagation, ray tracing*

1. Introduction

The acoustic waves essentially confined in jets, called neutral acoustic waves or guided jet waves (GJW), have been shown to play a major role in the development of instability waves and coherent structures and the establishment of resonance in jet flows [1]. Their properties have first been described using linear stability analyses [2], [3], providing their dispersion relations and their eigenfunction profiles in the radial directions. They have been found to strongly vary with the jet Mach number. More recently, GJW have also been investigated by solving problems of sound propagation in presence of a sheared flow [4], [5]. These studies have revealed that GJW can simply result from the total reflection of acoustic waves generated in the core of a jet flow by the shear layers.

In the present work, following the studies mentioned above, the formation of GJW is examined by tracing acoustic rays from a point source on the axis of a jet flow at a Mach number ranging up to 2. Despite ray tracing is only valid for high frequencies, it was successfully used in the past [6], [7] to calculate, for a source located in a jet, acoustic rays trapped in the jet potential core. Therefore, the objective here will be to compute guided jet rays (GJR) and to determine their main properties as a function of their initial angle and of the jet Mach number. These properties include the direction of propagation and the velocity of the rays, and their entire or partial confinement to the flow, obtained when the rays are fully contained inside the jet and when they occasionally pass through the outer part of the jet shear layers, respectively. The results will be compared with the characteristics of the GJW already known. Hopefully, they will bring additional insight into the mechanisms of formation of the acoustic waves guided in jets.

2. Parameters

Isothermal two-dimensional jets at Mach numbers ranging up to $M_j = u_j/c_j = 2$, where u_j and c_j are the velocity and speed of sound on the jet axis, are considered. They consist of cylindrical shear layers, surrounded by a medium at rest at temperature $T_a = 293$ K and pressure $p_a = 10^5$ Pa with speed of sound c_a ($c_j = c_a$). The profile of the axial velocity u_x is defined, using Cartesian coordinates (x, y), by the hyperbolic-tangent profile [8]

$$u_x(y) = \frac{u_j}{2}\left\{1 + \tanh\left[\frac{r_0}{4\delta_\theta}\left(\frac{r_0}{|y|} - \frac{|y|}{r_0}\right)\right]\right\} \quad (1)$$

where r_0 is the jet half-width such as $u_x(y = \pm r_0) = u_j/2$ and δ_θ is the shear-layer momentum thickness set to $\delta_\theta = 0.20 r_0$, as in a previous study [5], resulting in a thick shear layer. Velocity u_y is equal to zero, the jet pressure is equal to p_a and temperature is determined by a Crocco-Busemann relation.

A point source emitting rays at initial angles θ_0 relative to the jet direction varying between 0 and 360° in increment of 0.1° is introduced on the jet axis at $(x = 0, y = 0)$. The paths of the rays as they propagate

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away from the source are calculated by integrating the ray equations [6] given by

$$\begin{cases} \dfrac{dx_i}{dt} = c_0 \dfrac{k_i}{k} + u_{0i} \\ \dfrac{dk_i}{dt} = -k \dfrac{\partial c_0}{\partial x_i} - k_j \dfrac{\partial u_{0j}}{\partial x_i} \end{cases} \tag{2}$$

using an explicit fourth-order Runge-Kutta algorithm, where $(x_1, x_2) = (x, y)$ is the position along a ray, $k = (k_1, k_2)$ is the wave number, c_0 is the sound speed, and $u_0 = (u_{01}, u_{02}) = (u_x, u_y)$ is the flow velocity. The rays are traced for a time of $500 r_0/c_a$ using a time step equal to $\Delta t = 4 \times 10^{-5} r_0/c_a$. Their positions are recorded every $0.02 r_0/c_a$.

3. Results

The acoustic rays obtained at $M_j = 0.75$, 0.95 and 1.10 for emission angles varying between 0 and 360° every 1.5° are represented in figures 1(a-c). For the three Mach numbers, the rays initially directed downstream all leave out the flow and then propagate in the surrounding fluid at rest. In contrast, some of the rays initially directed upstream are guided by the jet for emission angles close to 180°. The GJR all travel in the upstream direction for $M_j = 0.75$, whereas they can travel upstream or downstream for $M_j = 0.95$ and 1.10. This behaviour is consistent with the group velocities of the GJW obtained using linear stability analyses [2], [3], which are always negative for low subsonic Mach numbers, but can be negative or positive for high subsonic and supersonic Mach numbers. In the same way and using the same terminology as for GJW [9], two types of GJR, namely duct-like (DL) and free-stream (FS) GJR, can be distinguished depending on whether the ray paths cross the shear-layer centerline at $|y| = r_0$. The duct-like GJR are confined to $|y| < r_0$, whereas the free-stream GJR propagate in the outer part of the shear layers between two successive passages through the jet potential core. The first ones appear for instance as light blue rays in figures 1(b,c), while the second ones are well visible upstream in figure 1(c).

To characterize the GJR obtained for the three Mach numbers, the velocities of the rays in the axial direction are plotted in figures 2(a-c) as a function of their emission angles. To highlight the nature of the GJR, the upstream-propagating free-stream and duct-like GJR are shown in blue and cyan, and the downstream-propagating free-stream and duct-like GJR are in red and magenta. GJR are observed between the critical angles θ_c for total internal reflection (in green) predicted using the laws of geometrical acoustics [10], [11]. This suggests that the angular ranges over which GJR can exist do not depend on the shear-layer thickness. Near the limit angles, in the three cases, the GJR are upstream-propagating free-stream GJR. Their velocities vary significantly and nearly reach the ambient speed of sound near for $\theta_0 \simeq \theta_c$, although they are guided by the jet and periodically cross the jet potential core. In addition to these GJR, the other GJR are upstream-propagating duct-like GJR for $M_j = 0.75$ in figure 2(a), upstream- and downstream-propagating duct-like GJR for $M_j = 0.95$ in figure 2(b), and downstream-propagating free-stream and duct-like GJR for $M_j = 1.10$ in figure 2(c). The velocity of the rays for $\theta_0 = 180°$ is equal to $c_j - u_j$ in all cases. Moreover, for $M_j = 0.95$ and 1.10, GJR characterized by zero velocity in the axial direction appear (in brown). They may be related to the GJW with zero group velocity (ZGV) [5] whose energy does not propagate axially, that are possible at such Mach numbers.

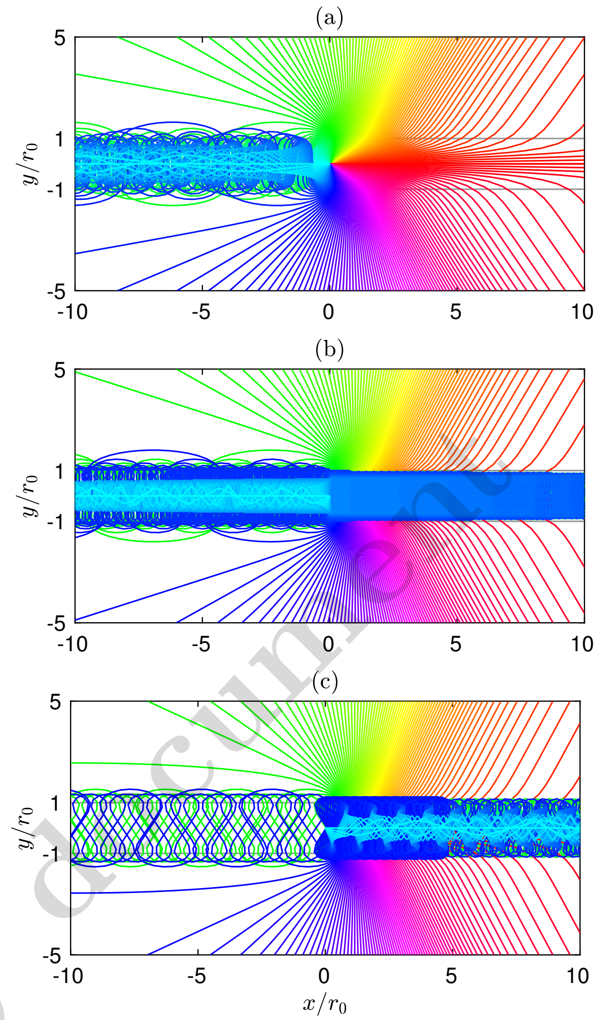

Figure 1: Acoustic rays obtained at (a) $M_j = 0.75$, (b) $M_j = 0.95$ and (c) $M_j = 1.10$ for θ_0 varying from 0 to 360° in increment of 1.5°; ——— $y = \pm r_0$.

The paths of typical GJR are illustrated in figure 3 for $M_j = 0.95$ and in figure 4 for $M_j = 1.10$. Those of the upstream-propagating free-stream GJR are traced in dark and medium blue. These rays all travel upwards in the outer part of the shear layer but downwards in the jet core. This explains how upstream-propagating GJR can exist and, counterintuitively, propagate against the flow even for supersonic Mach numbers. This also clarifies why, in such

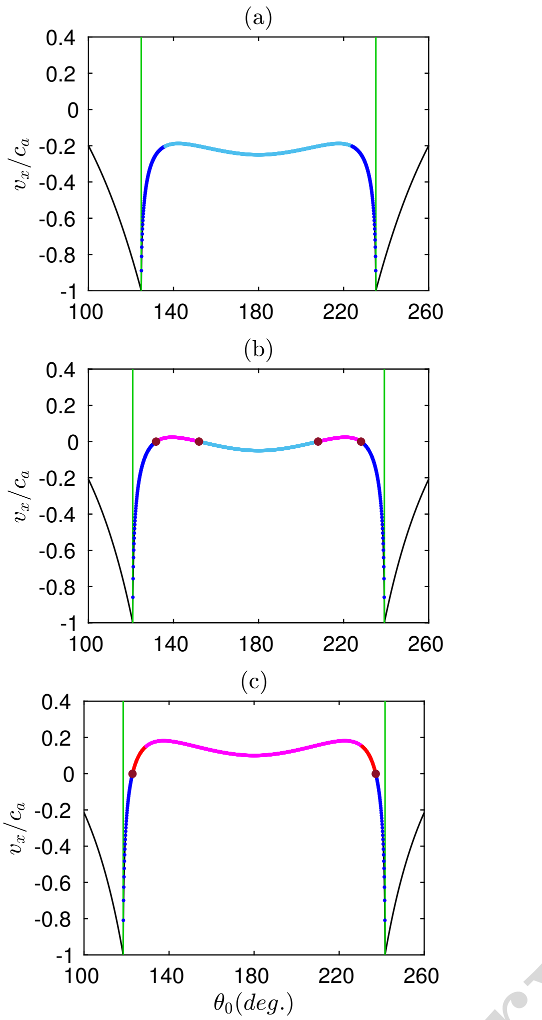

Figure 2: Ray velocity in the axial direction at (a) $M_j = 0.75$, (b) $M_j = 0.95$ and (c) $M_j = 1.10$ as a function of θ_0; upstream-propagating • free-stream and • duct-like GJR; downstream-propagating • free-stream and • duct-like GJR; • non-propagating GJR; —— limit angles for GJR according to geometrical acoustics.

cases, they must necessarily be of free-stream nature. Additionally, the paths of the upstream-propagating free-stream GJR lie outside the shear layers over sections of varying length, e.g. for the rays for $\theta_0 = 121.8°$ and $\theta_0 = 123.9°$ in figure 3. This leads to the significant disparities in their axial velocity reported above. More precisely, as the rays propagate more often at $|y| > r_0$, where the flow velocity is low, than within the jet, their axial velocity increases and approaches the ambient speed of sound. The paths of the GJR which does not propagate axially are shown in brown. They describe a closed, symmetrical figure-eight loop intersecting at the source point. This reveals the mechanism by which stationary waves can form in a jet flow through radial resonance. Lastly, the difference between free-stream and duct-like GJR appears clearly. As they propagate, they are reflected back in the jet potential core in the outer and the inner parts of the shear layers, respectively.

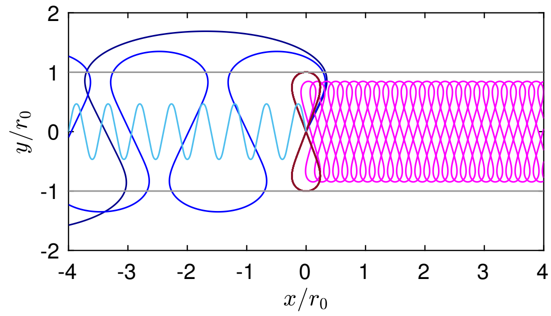

Figure 3: Rays at $M_j = 0.95$ for $\theta_0 =$ —— 121.8°, —— 123.9°, —— 131.8°, —— 140° and —— 170°; —— $y = \pm r_0$.

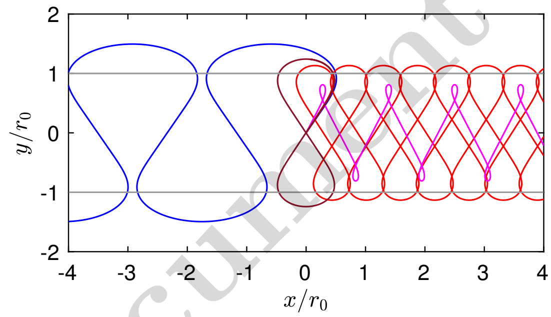

Figure 4: Rays at $M_j = 1.10$ for $\theta_0 =$ —— 120.2°, —— 122.8° —— 125° and —— 140°; —— $y = \pm r_0$.

Finally, considering all the jets examined in this study, the potential existence of rays guided in the jets, along with their nature, is illustrated in figure 5 as a function of the jet Mach number and the ray emission angle. The angular range over which GJR are observed widens with increasing Mach number. The GJR are found to all propagate upstream below a Mach number close to 0.92, but can travel upstream or downstream above. For all Mach numbers, most GJR exhibit duct-like behavior, except near the limit angles, where they transition to free-stream GJR. Among the free-stream GJR, the number of those propagating upstream is substantial for subsonic Mach numbers, but strongly decreases for supersonic ones. The latter trend can be connected to the narrowing of the allowable frequency ranges for upstream-propagating free-stream GJW predicted by linear stability analyses [9].

4. Conclusion

In this study, ray tracing was performed from an acoustic source in two-dimensional isothermal jets across a broad range of subsonic and supersonic Mach numbers. In every case, some rays initially emitted at small angles relative to the upstream direction are observed to be guided by the jets, and to be fully or partially confined in the jet flow. Their propagation directions, mean axial velocities, and free-stream or duct-like behaviors, depending on whether their paths extend into the outer regions of the jet shear layers, are determined as functions of the jet Mach number

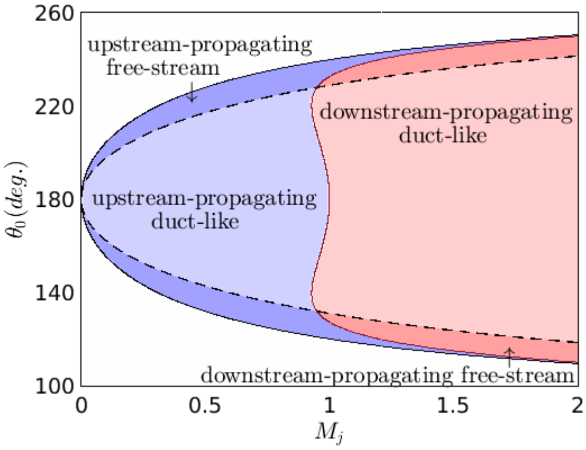

Figure 5: Nature of GJR as a function of M_j = and θ_0: upstream-propagating (blue) free-stream and (light blue) duct-like GJR; downstream-propagating (red) free-stream and (pink) duct-like GJR; ——— non-propagating GJR.

and the ray emission angle.
Although ray tracing is a simplified model based on a high-frequency approximation of sound propagation, the results are consistent with the known characteristics of guided jet waves in jets. For example, it is found that GJR can propagate downstream at high subsonic Mach numbers, similar to GJW. Furthermore, the results provides a clearer picture of the the formation mechanisms of these waves. For instance, they help us to explain how GJW can propagate upstream at nearly the ambient speed of sound, how GJW can travel upstream even in supersonic jets, and how stationary non-propagating GJW can form due to radial resonance.
Further insights could be obtained in future studies using the same methodology based on ray tracing by considering other jet flow profiles and other positions for the acoustic source.

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