

UNIFIED HIGH-ORDER 3D FINITE-ELEMENT FRAMEWORK FOR VIBROACOUSTIC FREQUENCY-RESPONSE ANALYSIS

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Abstract

A unified three-dimensional high-order finite-element framework is proposed for the frequency-response analysis of finite layered acoustic systems under plane-wave and diffuse acoustic field (DAF) excitation. The formulation relies on factorized high-order expansions of the Carrera Unified Formulation (CUF) type. At the same time, the present benchmark focuses on acousticized structures, namely solid layers recast as equivalent acoustic media with density inherited from the parent solid and longitudinal wave speed inferred from the elastic modulus. The same hierarchical discretization machinery is therefore used for fluid layers, equivalent acoustic solids, and perfectly matched layers. A transfer-matrix model is implemented both for classical plane-wave loading and for a discrete DAF extension based on the same random plane-wave realization adopted at the finite-element inlet. Three hierarchical models, namely LE4/B2, LE9/B3, and LE16/B4, are assessed over 2000 Hz to 16 000 Hz for an air-aluminum-air configuration; the DAF extension highlights the role of full three-dimensional modeling and outlet absorption. The study extends recent vibroacoustic literature based on CUF from eigenanalysis and wave propagation to the prediction of forced transmission loss in finite acousticized structures.

Keywords: *high-order finite elements; Carrera Unified Formulation; acoustic frequency response; diffuse acoustic field; transfer matrix method*

1. Introduction

Layered acoustic treatments, resonant media, and metamaterial-inspired concepts increasingly require simulation tools capable of spanning strong thickness gradients, finite-domain effects, and broadband excitation scenarios [1], [2], [3], [4]. In parallel, wave-based and reduced-order approaches have shown their value

for periodic porous media and optimized acoustic packages [5], [6], [7]. Within structural mechanics, the Carrera Unified Formulation (CUF) provides a hierarchical setting in which the approximation order is treated as a free parameter [8], [9]. Recent CUF studies have addressed wave propagation in two-dimensional waveguides and prestressed media [10], [11], [12], and a fully three-dimensional coupled structural-acoustic eigenanalysis has also been reported [13]. Forced acoustic transmission problems under external plane-wave or diffuse acoustic field (DAF) loading remain, however, comparatively unexplored in a fully three-dimensional factorized high-order setting.

The present work addresses this gap through a unified high-order finite-element framework for the frequency-response analysis of acousticized finite structures. The methodological contribution is threefold. First, solid layers are automatically converted into equivalent acoustic media while preserving the same high-order factorized mesh and interpolation hierarchy used for ordinary fluids. Second, both a single incident plane wave and a DAF represented as a superposition of random plane waves are handled through the same inlet-operator strategy. Third, the transfer matrix method (TMM) is extended to a DAF-consistent discrete form that uses the same angular realization and weights as the finite-element (FE) model. The resulting comparison is therefore stricter than a conventional FE/TMM comparison, because differences can be attributed primarily to dimensionality and boundary treatment rather than to inconsistent diffuse-field quadratures. The paper first presents the FE formulation, then summarizes the TMM benchmark, and finally discusses the three hierarchical simulations derived from the attached input files and result tables.

2. Unified high-order FE framework

2.1. Factorization and acousticization

For completeness, the problem is formulated for generic coupled structural-acoustic problems. In compact form, the harmonic finite-element equations can be written as

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$$(\mathbf{K}_c - \omega^2 \mathbf{M}_c + i\omega \mathbf{C}_c) \mathbf{q} = \mathbf{f}, \quad (1)$$

where $\mathbf{q} = [\mathbf{u}^T \mathbf{p}^T]^T$ is the coupled vector of structural displacements $\mathbf{u}$ and acoustic pressures $\mathbf{p}$, $\mathbf{K}_c$ is the global coupled stiffness matrix, $\mathbf{M}_c$ is the global coupled mass matrix, $\mathbf{C}_c$ is the damping matrix, and $\mathbf{f}$ is the generalized load vector. In block form, $\mathbf{K}_c$ contains the structural, acoustic, and coupling submatrices $(\mathbf{K}_{uu}, \mathbf{K}_{pp}, \mathbf{K}_{up}, \mathbf{K}_{pu})$, whereas $\mathbf{M}_c$ contains the structural and acoustic inertia submatrices $(\mathbf{M}_{uu}, \mathbf{M}_{pp})$. The present paper considers the pressure-only specialization obtained after acousticization of the central solid layer. Let $(\xi, \eta, \zeta) \in [-1, 1]^3$ denote the parent-element coordinates, where ξ and ζ span the in-plane section and η spans the thickness direction. In the present implementation, the three-dimensional approximation is factorized into a two-dimensional in-plane basis and a one-dimensional thickness basis,

$$\mathcal{N}_a(\xi, \eta, \zeta) = F_\alpha(\xi, \zeta) N_\beta(\eta), \quad (2)$$

where F_α is the α -th in-plane expansion, N_β is the β -th thickness expansion, and the compound index a identifies the pair (α, β) . In the Lagrange hierarchy used here, LE4, LE9, and LE16 correspond to $n_{px} = n_{py} = 2, 3$, and 4, whereas B2, B3, and B4 correspond to $n_{pz} = 2, 3$, and 4. The total number of interpolation points per element is thus

$$n_e = n_{px} n_{py} n_{pz}, \quad (3)$$

where n_{px} , n_{py} , and n_{pz} are the interpolation orders along x , y , and z , respectively. Although the adopted theoretical framework also supports Taylor in-plane expansions, only the Lagrange family is benchmarked herein. For a pressure-only acoustic element,

$$p^{(e)}(\mathbf{x}, \omega) = \sum_{a=1}^{n_e} \mathcal{N}_a(\mathbf{x}) \hat{p}_a^{(e)}(\omega), \quad (4)$$

where $p^{(e)}$ is the element pressure field, $\mathbf{x}$ is the physical position vector, ω is the angular frequency, and $\hat{p}_a^{(e)}$ is the nodal pressure unknown. In the present study, the central solid layer is acousticized, namely, converted into an equivalent acoustic medium with

$$\rho_a = \rho_s, \quad c_a = \sqrt{\frac{E_{\text{eff}}}{\rho_s}}, \quad (5)$$

where ρ_s is the parent-solid density, ρ_a is the equivalent acoustic density, E_{eff} is the effective longitudinal modulus, and c_a is the equivalent acoustic wave speed. For the aluminum benchmark, $E_{\text{eff}} = E = 70$ GPa and $\rho_s = 2700 \text{ kg m}^{-3}$, which gives $c_a \approx 5091.75 \text{ m s}^{-1}$.

2.2. Element matrices, inlet loading, and boundary conditions

The acoustic element matrices follow from the weak form of the Helmholtz problem:

$$\begin{aligned} K_{ab}^{(e)} &= \int_{\Omega_e} \frac{1}{\rho} \nabla \mathcal{N}_a \cdot \nabla \mathcal{N}_b \, d\Omega, \\ M_{ab}^{(e)} &= \int_{\Omega_e} \frac{1}{K_b} \mathcal{N}_a \mathcal{N}_b \, d\Omega, \end{aligned} \quad (6)$$

where Ω_e is the element volume, ρ is the local density, $K_b = \rho c^2$ is the bulk modulus, and c is the local sound speed. After assembly, node merging, and boundary-condition elimination, the reduced frequency-domain system becomes

$$\left[\mathbf{K} + \mathbf{K}_{\text{PML}}(\omega) - \omega^2 (\mathbf{M} + \mathbf{M}_{\text{PML}}(\omega)) + i \frac{k_0}{\rho_0} \mathbf{S}_\Gamma \right] \mathbf{p} = \mathbf{f}_\Gamma, \quad (7)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the assembled physical matrices, $\mathbf{K}_{\text{PML}}$ and $\mathbf{M}_{\text{PML}}$ are the Perfectly Matched Layer (PML) contributions, $\mathbf{S}_\Gamma$ is the inlet boundary operator, $\mathbf{p}$ is the vector of free pressure unknowns, $k_0 = \omega/c_0$ is the incident-fluid wavenumber, c_0 is the incident-fluid sound speed, and ρ_0 is the incident-fluid density.

For a single incident plane wave of amplitude p_0 and unit direction $\hat{\mathbf{k}}$,

$$p^{\text{inc}}(\mathbf{x}, \omega) = p_0 \exp \left[-ik_0 \hat{\mathbf{k}} \cdot (\mathbf{x} - \mathbf{x}_{\text{ref}}) \right], \quad (8)$$

where $\mathbf{x}_{\text{ref}}$ is a reference point on the inlet plane. The corresponding excitation vector is

$$f_{\Gamma,a} = \frac{2ik_0}{\rho_0} p_0 \int_{\Gamma_{\text{in}}} \mathcal{N}_a \exp \left[-ik_0 \hat{\mathbf{k}} \cdot (\mathbf{x} - \mathbf{x}_{\text{ref}}) \right] d\Gamma, \quad (9)$$

where Γ_{in} is the inlet boundary and $f_{\Gamma,a}$ is the a -th load component. The DAF is represented as a finite superposition of N_{PW} plane waves,

$$p_{\text{DAF}}^{\text{inc}}(\mathbf{x}, \omega) = \sum_{j=1}^{N_{\text{PW}}} p_0 w_j \exp \left[-ik_0 \hat{\mathbf{k}}_j \cdot (\mathbf{x} - \mathbf{x}_{\text{ref}}) \right], \quad (10)$$

where $\hat{\mathbf{k}}_j$ is the propagation direction of wave j and w_j is its complex weight. In the present study, $N_{\text{PW}} = 64$, the directions are sampled over the inward hemisphere with a cosine-weighted law, and the weights satisfy $\sum_{j=1}^{N_{\text{PW}}} |w_j|^2 = 1$.

The incident power associated with the discrete DAF realization over the inlet area A_{in} is

$$P_{\text{inc}}^{\text{DAF}} = \frac{A_{\text{in}}}{2\rho_0 c_0} \sum_{j=1}^{N_{\text{PW}}} |p_0 w_j|^2 \cos \theta_j, \quad (11)$$

where θ_j is the angle between the inward inlet normal and direction $\hat{\mathbf{k}}_j$. A terminal PML is appended to the outlet fluid and is described by the polynomial complex stretch



$$s_n(\chi, \omega) = \kappa + i \frac{\sigma_n(\chi)}{\omega}, \quad \sigma_n(\chi) = \sigma_{\max} \left(\frac{\chi}{L_{\text{PML}}} \right)^m, \quad (12)$$

where χ is the local coordinate along the outlet normal, κ is the real stretch factor, L_{PML} is the PML thickness, m is the polynomial order, and $\sigma_{\max}$ is selected from a target normal-incidence reflection level according to

$$\sigma_{\max} = -\frac{(m+1)c \ln R_t}{2L_{\text{PML}}}, \quad (13)$$

where c is the local sound speed and R_t is the target normal-incidence reflection coefficient [14]. Acoustic powers are extracted from

$$P_\Gamma = \frac{1}{2} \Re \left(\int_\Gamma p \overline{v_n} d\Gamma \right), \quad v_n = -\frac{1}{i\omega\rho} \frac{\partial p}{\partial n}, \quad (14)$$

where P_Γ is the net power through control surface Γ , v_n is the normal particle velocity, and $\partial p / \partial n$ is the pressure derivative along the outward unit normal. The transmission loss is finally

$$TL(\omega) = 10 \log_{10} \left(\frac{P_{\text{inc}}(\omega)}{P_{\text{tra}}(\omega)} \right), \quad (15)$$

where P_{inc} and P_{tra} are the incident and transmitted acoustic powers.

3. Transfer matrix benchmark

For stratified acoustic media, transfer-matrix descriptions remain a standard and computationally efficient benchmark for reflection and transmission analyses, and stable multilayer formulations have been developed precisely to avoid the numerical ill-conditioning that may affect classical implementations [15]. Related developments have also extended transfer-matrix ideas to periodic planar media by deriving through-thickness transfer operators directly from finite-element unit-cell models, thereby strengthening the connection between wave-based reduced descriptions and high-fidelity discretizations [16]. For a stratified stack, the classical TMM relates pressure and normal velocity across each layer. For layer ℓ ,

$$\mathbf{T}_\ell = \begin{bmatrix} \cos(k_{n,\ell} d_\ell) & iZ_{n,\ell} \sin(k_{n,\ell} d_\ell) \\ iZ_{n,\ell}^{-1} \sin(k_{n,\ell} d_\ell) & \cos(k_{n,\ell} d_\ell) \end{bmatrix}, \quad (16)$$

where d_ℓ is the layer thickness, $k_{n,\ell}$ is the normal wavenumber, and $Z_{n,\ell}$ is the normal acoustic impedance. If k_ℓ is the total wavenumber and k_t is the tangential component,

$$k_{n,\ell} = \sqrt{k_\ell^2 - k_t^2}, \quad Z_{n,\ell} = Z_{c,\ell} \frac{k_\ell}{k_{n,\ell}}, \quad (17)$$

where $Z_{c,\ell}$ is the characteristic impedance. Let $\mathbf{T} = \prod_{\ell=1}^{N_L} \mathbf{T}_\ell$ be the transfer matrix of the finite stack, where N_L is the number of internal finite layers. If $Z_{0,n}$

and $Z_{L,n}$ are the normal impedances of the incident and transmitted half-spaces, one defines

$$A = T_{11} + \frac{T_{12}}{Z_{L,n}}, \quad B = T_{21} + \frac{T_{22}}{Z_{L,n}}, \quad (18)$$

and obtains

$$r = \frac{A/Z_{0,n} - B}{A/Z_{0,n} + B}, \quad t = \frac{1+r}{A}, \quad \tau = \frac{\Re(Z_{L,n})}{\Re(Z_{0,n})} |t|^2, \quad (19)$$

where r and t are the complex reflection and transmission coefficients and τ is the transmitted-to-incident power ratio. For a rigid backing, $Z_{L,n} \rightarrow \infty$ and $\tau = 0$.

The discrete DAF extension introduced here uses the same plane-wave realization as the FE inlet. For plane wave j ,

$$k_{t,j} = k_0 \sin \theta_j, \quad (20)$$

where θ_j is the incidence angle measured from the inlet normal. The diffuse-field averaged reflectance and transmittance are then

$$\begin{aligned} \bar{R} &= \frac{\sum_{j=1}^{N_{\text{PW}}} |w_j|^2 \cos \theta_j R_j}{\sum_{j=1}^{N_{\text{PW}}} |w_j|^2 \cos \theta_j}, \\ \bar{\tau} &= \frac{\sum_{j=1}^{N_{\text{PW}}} |w_j|^2 \cos \theta_j \tau_j}{\sum_{j=1}^{N_{\text{PW}}} |w_j|^2 \cos \theta_j}, \end{aligned} \quad (21)$$

where $R_j = |r_j|^2$ is the reflectance of wave j . The DAF-consistent transmission loss follows from

$$TL_{\text{DAF}} = -10 \log_{10}(\bar{\tau}). \quad (22)$$

4. Numerical campaign

The benchmark is an air-aluminum-air stack aligned with the y axis. The bottom air region is 150 mm thick and rigidly backed. The central aluminum layer is 10 mm thick and is acousticized according to Eq. (5). The top air region is again 150 mm thick and is prolonged by an automatically generated PML cap. The lateral dimensions are 150 mm in x and 100 mm in z . The incident field enters from $y_{\min}$ with amplitude $p_0 = 1$ Pa; for the plane-wave case, the incidence is normal, whereas the DAF uses 64 inward plane waves, cosine-weighted and random-phased. The frequency sweep covers 2000 Hz to 16 000 Hz with 100 samples. The three input decks differ only in polynomial order and macro-discretization, so the comparison isolates the numerical role of hierarchical enrichment. Table 1 collects the physical parameters and solver settings, whereas Fig. 1 summarizes the layered configuration. The three tested discretizations only differ in polynomial order and macro-mesh density. Their main characteristics are reported in Table 2. A noteworthy practical aspect is that the highest-order model, LE16/B4, also has the fewest global pressure unknowns because p-enrichment is offset by macro-mesh coarsening. This

Table 1: Physical model and analysis settings common to all runs.

Item	Value
Lateral dimensions	150 mm × 100 mm
Layer thicknesses	150 mm air / 10 mm acousticized aluminum / 150 mm air
Incident-fluid properties	$\rho_0 = 1.2 \text{ kg m}^{-3}$, $c_0 = 343 \text{ m s}^{-1}$
Equivalent aluminum properties	$\rho_a = 2700 \text{ kg m}^{-3}$, $c_a = 5091.75 \text{ m s}^{-1}$
Excitation	Plane wave or 64-wave DAF, $p_0 = 1 \text{ Pa}$
Sweep	2000 Hz to 16 000 Hz, 100 points
Termination PML target reflection	bottom rigid, top PML $R_t = 10^{-6}$

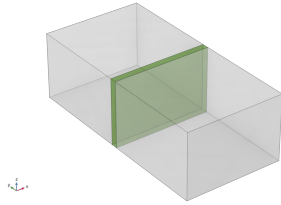


Figure 1: Benchmark configuration. The original structural layer is acousticized and solved as a pressure-only equivalent acoustic medium, while an automatically generated PML cap extends the outer top fluid.

behavior is a direct consequence of the factorized high-order setting.

5. Results and discussion

Figs. 2a to 2c compare the transmission loss curves from the TMM, the external COMSOL FE reference, and the proposed CUF-based FE solver. Under plane-wave excitation, the agreement between CUF and TMM is excellent across the physically relevant part of the band. The mean absolute discrepancy remains below 0.42 dB for all three CUF hierarchies. The comparison with COMSOL is also satisfactory, with discrepancies close to 1.1 dB over the same frequency range. Localized upper-end spikes are observed for LE9/B3 around 17.8 kHz and for LE16/B4 around 19.4 kHz; these isolated features do not alter the overall trend and are interpreted as conditioning-sensitive phenomena linked to sparse sampling at the top of the band rather than to a systematic inconsistency of the formulation.

The DAF case is more revealing because it highlights the difference between a one-dimensional benchmark and a finite three-dimensional model. The CUF/TMM mean absolute discrepancy lies between 2.37 dB and

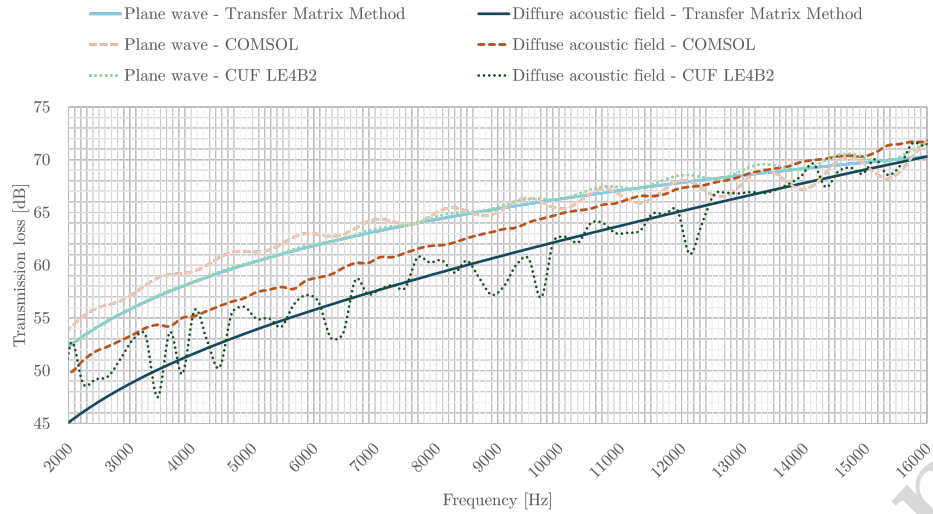
2.62 dB. The CUF/COMSOL discrepancy is somewhat larger, yet t. Yet, the three-dimensional simulations reproduce the overall DAF trend more faithfully than a purely one-dimensional model would, precisely because they explicitly account for the imposed angular realization, the finite lateral extent, and the outlet PML. Around 1 kHz, for instance, the CUF DAF transmission loss is approximately 49.3 dB for all hierarchies, which is much closer to the COMSOL values near 50 dB than to the one-dimensional TMM estimate near 39.4 dB. The most severe discrepancy occurs at the lowest sampled frequency, where the COMSOL diffuse-field transmission loss exceeds 77 dB while both CUF and TMM predictions remain much lower. Since CUF and TMM remain mutually consistent at that point, this difference is interpreted as a low-frequency implementation-sensitive artifact of the external DAF/PML setup rather than as a physical feature of the layered stack itself.

From a methodological standpoint, the DAF results demonstrate the utility of a unified three-dimensional frequency-response framework even for nominally stratified configurations. Once the excitation is no longer restricted to a single normal-incidence plane wave, the response becomes sensitive to angular content, finite-size effects, and outlet absorption. At the same time, the comparison among LE4/B2, LE9/B3, and LE16/B4 confirms that p-enrichment combined with macro-mesh coarsening can maintain the overall response quality without increasing the number of unknowns. The LE16/B4 model is particularly illustrative in this respect, as it reduces the active pressure degrees of freedom by approximately half compared to the lower-order cases while still reproducing the broad-band response accurately, except for a very localized upper-band spike.

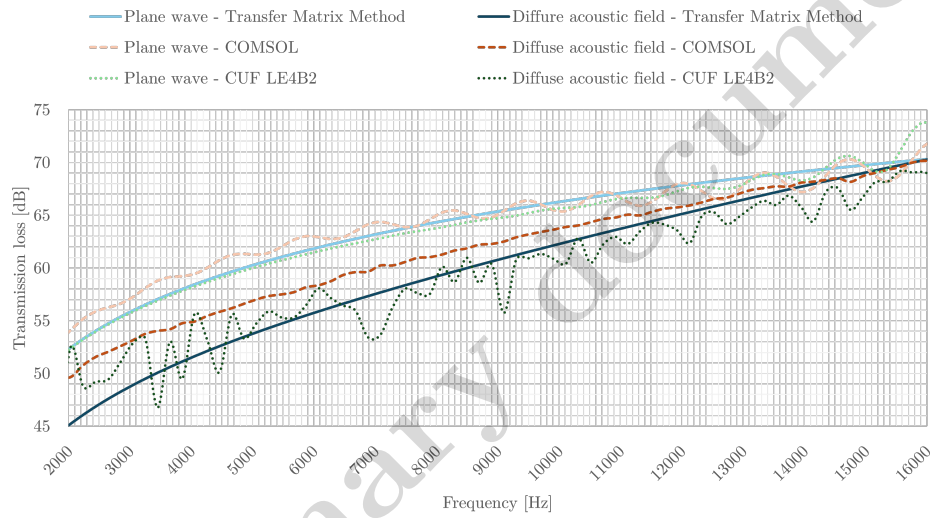
6. Conclusions

A unified high-order three-dimensional CUF framework for acoustic frequency-response analysis has been presented. The formulation extends factorized CUF-type ideas from modal and wave-propagation settings to forced transmission loss prediction under plane-wave and diffuse-field loading. The central modeling idea is the acousticization of solid layers, which allows finite fluids, equivalent acoustic solids, and PML domains to be treated inside a single pressure-based solver.

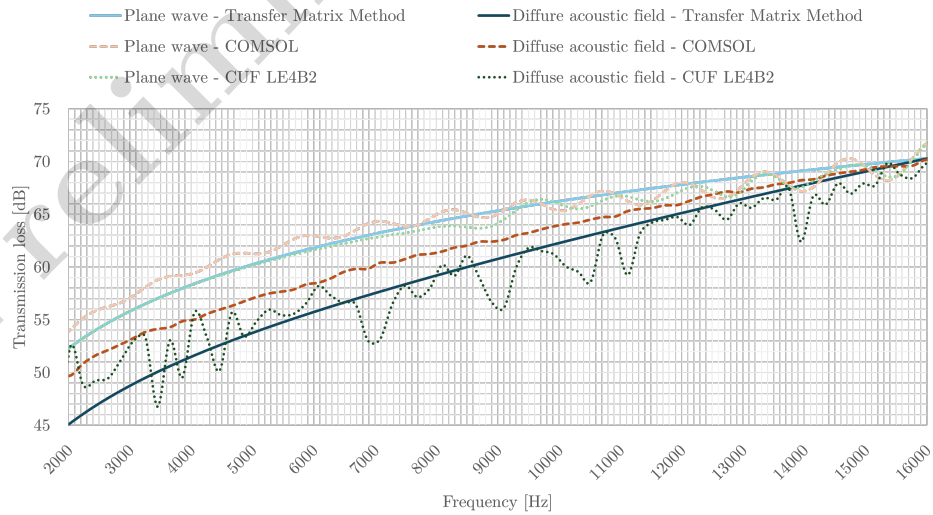
The numerical results show that the proposed CUF framework reproduces plane-wave TMM predictions with excellent accuracy over a broad frequency range. At the same time, the DAF-consistent comparison highlights the need for full three-dimensional modeling once angular loading and absorbing outlets are introduced. The next developments should focus on fully coupled displacement-pressure frequency-response analyses of orthotropic, porous, and poroelastic layers under diffuse acoustic fields, and on optimization workflows for layered acoustic treatments and metamaterial-inspired concepts.



(a) LE4/B2.



(b) LE9/B3.



(c) LE16/B4.

Figure 2: Comparison of transmission loss curves from the TMM, the external COMSOL FE reference, and the proposed CUF-based FE solver.

Table 2: Hierarchical discretizations.

Order	Fluid mesh	Layer mesh	3D elements	Degrees of freedom
LE4/B2	30x30x20	30x2x20	64,200	70,308
LE9/B3	15x15x10	15x1x10	7,950	69,657
LE16/B4	8x8x5	8x1x5	1,160	35,200

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