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INCREASING RELIABILITY IN TUBE MEASUREMENTS FOR CHARACTERIZATION OF POROUS MATERIALS

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Abstract

The three-microphone transfer function method has been a tube measurement method widely accepted for characteristic impedance and propagation coefficient of porous materials. When combined with the multiple-microphone method, the measurement frequency range can be extended to almost twice that of traditional measurements. However, discontinuities often emerge in the wavenumber spectrum at high frequencies. This problem is due to the inherent nature of the inverse cosine function. This work unwraps the complex-valued wavenumber spectrum with sequential Bayesian inference. The results demonstrate the reliability of the proposed method for broadband characterization of porous materials.

Keywords: *sequential Bayesian inference, impedance tube measurement, characteristic impedance*

1. Introduction

The impedance tube measurement with transfer function/matrix is one of the most popular measurement methods for characterization of acoustic materials since it was introduced [1], [2]. A three-microphone transfer function method was proposed to characterize the characteristic impedance and propagation coefficient [3]. However, this method still suffers from the constraints of the assumption of plane waves. Sanada [4] and Panzer [5] introduced the cylindrical decomposition to extend the frequency range. While the frequency range of measurement is extended, phase discontinuities occur in the measured propagation coefficient and characteristic impedance at high frequencies. The phase discontinuities come from the wrapped wavenumber spectrum, which is due to the inherent drawback of the inverse cosine function. This work applies a numerical inversion method to resolve the phase wrapping problem.

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2. Measurement method

This work combines the three-microphone transfer function method [3] and the multiple microphone method [5]. Multiple microphones are distributed evenly along the circumference at each longitudinal cross section. With the multiple microphone method, the circumferential modes smaller than the number of microphones at each cross section would be canceled. In this way, the valid frequency range for the measurement is extended to

$$f = 1.22\,\frac{c_0}{d}, \tag{1}$$

where c_0 is the sound speed and d is the diameter of the tube.

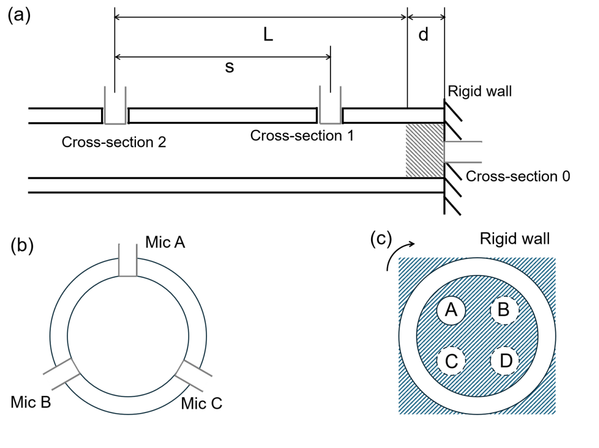

Figure 1: Sketch for the multiple microphone impedance tube measurement. (a)General setup for the whole tube; (b)Microphone positions at cross section 1 & 2; (c)Microphone positions at cross section 0.

The propagation coefficient of the tested material is determined by [3]

$$\underline{k} = \frac{1}{d}\cos^{-1}\underline{H}_{d0}, \tag{2}$$

where the transfer function $\underline{H}_{d0}$ is the ratio of the impulse response right in front of the tested material and the impulse response at back of the tested material in the frequency domain. The measurement of the impulse response right in the front center of the tested material requires a microphone placed in the center of the tube, which will interfere with the sound waves propagating inside the tube. Salissou [3] modified the measurement so that the microphone won't interfere with the sound waves. The complex-valued transfer function $\underline{H}_{d0}$ can be calculated as [3]

$$\underline{H}_{d0} = \frac{1+\underline{R}}{\mathrm{e}^{\underline{\gamma}(L-s)} + \underline{R}\mathrm{e}^{-\underline{\gamma}(L-s)}}\,\underline{H}_{10}, \tag{3}$$

where $\underline{H}_{10}$ is equal to the ratio of the impulse responses at cross sections 1 and 0, $\underline{\gamma}$ is the propagation coefficient of air [6], and $\underline{R}$ is the surface reflectance. Therefore, the characteristic impedance is determined by

$$\underline{Z}_c = \mathrm{j}\frac{1+\underline{R}}{1-\underline{R}}\tan(\underline{k}d). \tag{4}$$

3. Phase Unwrapping

The combination of the three-microphone transfer function method and multiple microphone method allows us to characterize the porous materials at higher frequencies (2.5kHz to 9.5 kHz). However, we observe the undesired discontinuities in the measured propagation coefficient and characteristic impedance (see Figure 2). The calculation of the characteristic impedance requires the propagation coefficient, and the propagation coefficient of the testing material is proportional to the inverse cosine of the transfer function. The inverse cosine of the transfer function is a multivalued function. For brevity, we define the wavenumber spectrum equal to the propagation coefficient times thickness $\underline{k}d$. The wavenumber spectrum is unknown, while the cosine value–the transfer function $\underline{H}_{d0}$–is known. Although the magnitude of the sine value of the wavenumber spectrum can be easily calculated, it can be either positive or negative. This sign ambiguity causes errors in the cosine inversion, leading to the phase discontinuities observed in the propagation coefficient and characteristic impedance. To resolve this problem, this work applies sequential Bayesian inference for the estimation of the unwrapped wavenumber spectrum.

3.1. Sequential Bayesian inference

The model M chosen for the Bayesian inference is the cosine value of the wavenumber spectrum. Since the model is not a function of frequency, we estimate the wavenumber spectrum at each frequency bin. The mean and variance of the estimated posterior at each data point are used as the mean and variance of the prior for the subsequent data point [7]. The measured propagation coefficient is reliable at low frequencies and should be continuous in the frequency domain. In addition. Based on our physical understanding, the sequential Bayesian inference framework to unwrap

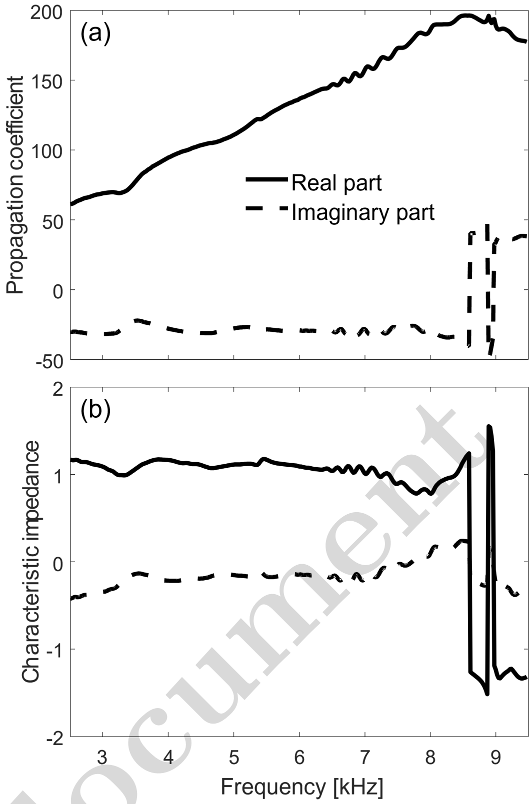

Figure 2: Measured propagation coefficient and characteristic impedance at an extended frequency range.

the data. Bayes' theorem for the estimation is

$$\overbrace{p(\boldsymbol{\theta}|\underline{H}_{\mathrm{D}},\underline{H}_{\mathrm{M}},I)}^{\text{posterior}} = \frac{\overbrace{p(\underline{H}_{\mathrm{D}}|\boldsymbol{\theta},\underline{H}_{\mathrm{M}},I)}^{\text{likelihood}} \times \overbrace{p(\boldsymbol{\theta}|\underline{H}_{\mathrm{M}},I)}^{\text{prior}}}{p(\underline{H}_{\mathrm{D}}|\underline{H}_{\mathrm{M}},I)}, \tag{5}$$

where θ collectively represents the parameter–that is the wavenumber spectrum $\underline{k}d$–, and I is the background information. In the sequential Bayesian inference framework, the prior distribution at each data point is derived from the posterior distribution of the previous point. As a result, the method inherently favors smooth and continuous variations in the data. The prior distribution is assigned to be a Gaussian distribution with a mean of the posterior from the previous point. The standard deviation is 1.2 times the standard deviation of the posterior for flexibility. The likelihood is assigned to be a Student-t distribution. [8]

$$p(\underline{H}_{\mathrm{D}}|\boldsymbol{\theta},\underline{H}_{\mathrm{M}},I) = \frac{\Gamma(K/2)}{2}\left(\pi\sum_{k=1}^{K}\epsilon_k^2\right)^{-K/2}, \tag{6}$$

where $\Gamma(\dots)$ is the standard Gamma function, K is the number of points–that is 1 in this case–, and ϵ_k is the residual error.

4. Results

The measurements were carried out in accordance with ASTM E1050-19 [9]. The impedance tube is constructed from PVC with a length of 2 m, a wall thickness of 6.4 mm, and an upper frequency limit of 5.4 kHz for the conventional transfer-function method. Figure 3 shows the microphone arrangement at cross section 0. To simplify fabrication while preserving the rigidity of the tube termination, only a single hole is drilled in the solid metal backing. This design ensures that the backing remains sufficiently rigid for accurate measurements. During the measurements, impulse responses at four circumferential positions on cross section 0 are acquired sequentially by rotating the metal backing together with the microphone.

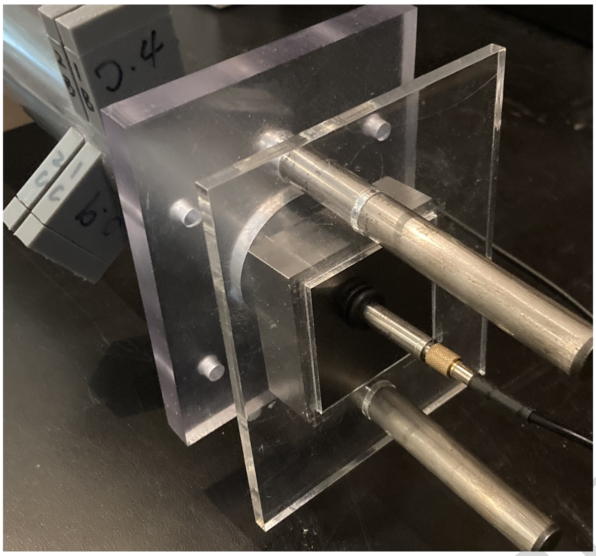

Figure 3: Photography for the microphone position at cross section 0.

The distance between cross section 1 and the test specimen is 10.2 cm, and the spacing between cross sections 1 and 2 is 1.2 cm. The measured propagation coefficient and transfer function are then used as inputs to the sequential Bayesian inference. During the estimation, 500 samples are generated at each frequency point. The complete inference process requires less than 1.5 s in MATLAB on a standard personal computer.

The estimation result is shown in Figure 4. One noticeable observation from the figure is that the ripples occur in both measured and estimated results at high frequencies. These ripples may be caused by manufacturing errors or uncertainties in the microphone mounting. The tested material in this work is a piece of hand-cut melamine foam, and the edge of the material is not smooth and ideal. In addition, a small off-position of the microphones, either on the longitudinal or circumferential axis, may introduce contributions from circumferential modes inside the tube. The microphone used in this measurement is a 1/4 inch microphone, and its surface is made to be

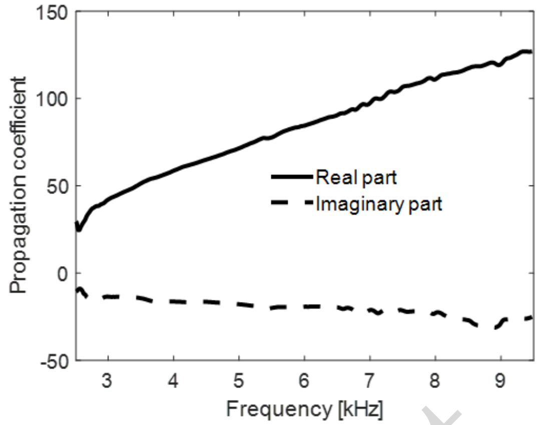

Figure 4: Unwrapped propagation coefficient through sequential Bayesian inference.

flush with the tube wall. There is still small surface roughness between the microphone and the tube wall. This roughness could cause the undesired behavior of sound waves at high frequencies.

5. Conclusion

This work demonstrates the challenge in the characterization of porous materials over an extended frequency range. The unexpected phase discontinuities occur at high frequencies due to the inherent sign ambiguity of the inverse cosine function. To resolve the problem, this work applies sequential Bayesian inference. By estimating each data point separately and propagating the posterior information forward as the prior knowledge, we adaptively unwrap the phase function. Future work includes calibrating the microphone mounting holes and using smaller microphones to reduce the uncertainties at high frequencies.

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