TIME OR FREQUENCY DOMAIN? CHOOSING MODELLING APPROACHES FOR SOUND PROPAGATION FROM SIMPLE MOVING SOURCES

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Abstract

When modelling sound radiation from moving sources such as passing trains or aircraft fly-overs, a key methodological choice is whether to work in the time or frequency domain. Although both approaches can be used to compute the received signal from monopole sources along arbitrary trajectories, their respective assumptions, computational costs, and suitability for different applications are not always straightforward. In this work, we systematically compare time-domain and frequency-domain formulations for moving monopole sources. We analyse their underlying assumptions, numerical efficiency, and accuracy across representative scenarios. The results provide practical criteria for selecting an appropriate approach depending on source motion and modelling objectives. In addition, we explore the integration of frequency-domain formulations into inverse methods for source reconstruction.

Keywords: *Moving sound sources, Time/Frequency-domain modelling, Numerical acoustics*

1. Introduction

Airplane flyovers and passing trains can create noise that seriously affects the nearby population. Therefore, it is crucial to reduce the noise generated by such events. To achieve this, we first need to understand which parts of planes and trains are responsible for the noise. This requires measuring and analyzing these objects while they are in motion. To interpret the signals from these moving sources, we need reliable methods of modeling this phenomenon. This paper addresses this issue.

This paper only considers the simplest source, which is a monopole. Airplanes and trains can be modeled as groups of these sources. Our focus is on modeling the propagation of sound from these simple, moving sources (see figure 1). More specifically, we aim to model the received acoustic pressure spectrum. There are two ways in which this problem has been solved. One method is to model the acoustic pressure time signal and then use a standard Fourier transformation to calculate the spectrum (i.e., the *time domain method*). The other method models the pressure spectrum directly in the frequency domain (i.e., the *frequency domain method*; see figure 2). The derivation of the received acoustic pressure time signal from moving monopoles has been described among others by Morse and Ingard [1], by Delfs [2] and by Rienstra and Hirschberg [3]. Ochmann and Piscoya [4] and Zhang et al. [5] have developed a frequency-domain method.

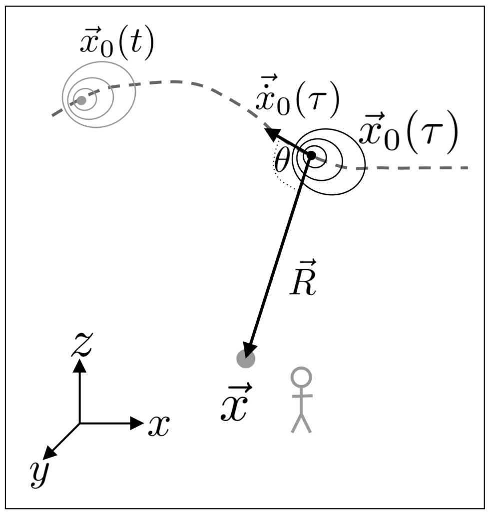

Figure 1: An illustration of the problem at hand. We aim to calculate the received acoustic pressure signal at an observation point $\vec{x}$, coming from a source, which moves along the path $\vec{x}_0$ at a speed $\dot{\vec{x}}_0$. We distinguish between the emission time τ of the signal at the source and its observation time t at the observer. R is the emission distance and θ the angle between $\vec{R}$ and $\dot{\vec{x}}_0$.

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Our aim is to compare these two methods of calculating the acoustic pressure spectrum for different use cases and identifying categories to inform the choice of method for each case. In Section 2, we present the theory behind the time and frequency domain methods. Section 3 presents a benchmark test to compare calculation times and verify that both methods produce the same results. Section 4 presents the results of the benchmark test. Section 5 investigates the frequency domain method's suitability for inverse problems.

2. Theory

Before we can calculate and compare the acoustic pressure spectra received from moving monopoles using the two different methods, we must first review the existing theory.

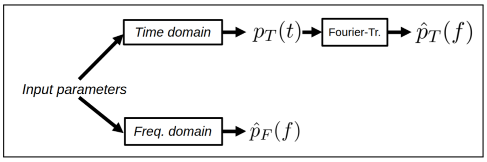

Figure 2: Flowchart showing the calculations in the time domain method (upper branch) and the frequency domain method (lower branch). The pressure time signal $p_T(t)$ is calculated using equation (5), its Fourier transform $\hat{p}_T(f)$ using equation (7) and the frequency domain method $\hat{p}_F(f)$ using equation (10).

Both the methods in the time and frequency domains start with the same wave equation. It describes the acoustic pressure field p that is generated by a monopole source moving along a given path $\vec{x}_0(t)$ [4, eq. (2.1)],[1, p. 721],[3, eq. (9.16)]. Here, the source is a heat or mass source with a strength $F(t)$ [2, p. 101][1, p. 721].

$$\begin{aligned}\Delta p(\vec{x},t) - \frac{1}{c^2}\frac{\partial^2 p(\vec{x},t)}{\partial t^2} &= \\ -\frac{\partial}{\partial t}\left[F(t)\delta(\vec{x}-\vec{x}_0(t))\chi_{[t_o,t_1]}(t)\right]&\end{aligned} \tag{1}$$

On the left-hand side of the wave equation (1), the symbol Δ denotes the Laplacian operator [4]. We have added the characteristic function $\chi_{[t_0,t_1]}(t)$ to indicate that we assume a finite observation time. It's value is 1 if $t \in [t_0, t_1]$ and 0 otherwise.[4, p. 42 f.]

2.1. Time domain

The wave equation (1) is more easily solved if we consider the acoustic potential field φ rather than the acoustic pressure field p, which is defined as follows [4, p. 10],[2, p. 101],[4, p. 10]:

$$p = -\rho \frac{\partial \varphi}{\partial t} \tag{2}$$

where ρ denotes the density of the medium. We use the Green's function method to solve the wave equation of the acoustic potential field φ [1, eq. (7.1.18)],[2, p. 101][3, p. 264].

From this we find the expression for the acoustic velocity potential.[3, eq. (9.18)][2, p. 102]:

$$\varphi(\vec{x},t) = \frac{F(t-R/c)}{4\pi R(1-M\cos\theta)} \tag{3}$$

with R the emission distance (see below) and $M = |\vec{M}| = |\dot{\vec{x}}_0(t-R/c)/c|$ the Mach-number. Here $\dot{\vec{x}}_0$ denotes the velocity vector of the source. Further, let θ be the angle between the vectors $\vec{R}$ and $\vec{M}$, i.e., $\cos\theta = \vec{R}\cdot\vec{M}/(RM)$. This solution is known as the Liénard-Wiechert potential [3, p. 264].

In order to propagate the emission signal to the receiver, we need to calculate the distance R between the source and the receiver at the emission time $\tau = t - R/C$. The emission time τ, describes the time at which the signal, that arrives at the receiver at the time t, is emitted from the source [2, p. 101],[3, p. 264 f.],[1, p. 718]. Here, c denotes the speed of sound in the medium.

Mathematically, we can describe R as $|\vec{R}| = |\vec{x} - \vec{x}_0(t-R/c)|$ [1, p. 718], with $\vec{x}$ the position of the observer and $\vec{x}_0(t) = (x_0(t), y_0(t), z_0(t))^T$ the arbitrary path of the point source. It follows that [1, p. 11.2.1]:

$$\begin{aligned}R^2 = &\left[x - x_0\left(t-\frac{R}{c}\right)\right]^2 + \left[y - y_0\left(t-\frac{R}{c}\right)\right]^2 + \\ &\left[z - z_0\left(t-\frac{R}{c}\right)\right]^2\end{aligned} \tag{4}$$

This equation can be solved analytically for a straight-line path at constant speed. However, for an arbitrary path, it must be solved numerically.

With the expression for the velocity potential (see equation (3)), the expression for the pressure is derived from equation (2) [2, eq. (242)][3, eq. (9.20)]:

$$\begin{aligned}p_T(t) = &\frac{\rho}{4\pi}\frac{F'(\tau)}{R(1-M\cos\theta)^2} + \\ &\frac{\rho F(\tau)}{4\pi}\frac{\vec{R}\cdot\dot{\vec{M}} + cM(\cos\theta - M)}{R^2(1-M\cos\theta)^3}\end{aligned} \tag{5}$$

This means that when calculating the pressure time signal $p_T(t)$ (see equation (5)), the emission distance R must be calculated for each observation time point t, by solving equation (4).

When comparing the results between the time- and the frequency domain methods, we will look at a single frequency source, described by the amplitude Q and the frequency f_0:

$$F(t) = Q\mathrm{e}^{-i2\pi f_0 t} \tag{6}$$

To calculate the received acoustic pressure spectrum, using the time domain method, we must perform the Fourier transform on the time signal of the received pressure $p_T(t)$ (see figure 2). Seeing as we have a factor (-1) in the exponent of equation (6), we will use following convention for the Fourier-Transform [4, eq. (2.4)]:

$$\hat{p}_T(f) := \mathcal{F}_t[p_T(t)] := \int_{-\infty}^{\infty} p_T(t)\mathrm{e}^{i2\pi ft}dt \quad (7)$$

Technically, we integrate for $t \in [t_0, t_1]$, where t_0 and t_1 are the observation time limits. For the initial implementation, we will calculate the integral in equation (7) using the trapezoidal rule.

2.2. Frequency domain

In this section, we derive the equation for the acoustic pressure spectrum $\hat{p}_F(f)$ directly in the frequency domain. Here, we also first consider the acoustic velocity potential. We denote the Fourier transform of the acoustic pressure by the symbol $\hat{p}$ and the Fourier transform of the acoustic velocity potential by $\hat{\varphi}$. It follows that [4, eq. (2.5)]:

$$\hat{p}(f) = i2\pi f\rho\hat{\varphi}(f) \quad (8)$$

Using the Fourier transform on the wave equation (1), the Helmholtz equation is derived [4, p. 14]:

$$\begin{aligned}&\Delta\hat{p}(\vec{x},\omega) + \left(\frac{\omega}{c}\right)^2\hat{p}(\vec{x},\omega) = \\ &i\omega\rho\int_{t_0}^{t_1} F(t)\delta(\vec{x}-\vec{x}_0(t))\mathrm{e}^{i\omega t}dt\end{aligned} \quad (9)$$

As in the time domain, the Green's function method is used to solve equation (9) for $\hat{p}$ [4, eq. (2.22)].

This yields the following expression for the acoustic pressure spectrum [5]:

$$\begin{aligned}\hat{p}_F(f) =& -i2\pi f \\ &\int_{\tau_0}^{\tau_1} \frac{F(\tau)\exp\{i2\pi f(\tau + R(\tau)/c)\}}{4\pi R(\tau)}d\tau\end{aligned} \quad (10)$$

The expression in equation (10) is equal to the Fourier transform of the Lienard-Wiechert potential (see equation (3)) after substituting $t \to \tau = t - R/c$ and adding the factor $i2\pi f$ to transform the potential into pressure (see equation (8)). We only consider the observation time interval $[t_0, t_1]$ to be known. To find the emission time limits, τ_0 and τ_1, we must solve the equation (4) for R twice numerically. We calculate the integral in equation (10) using the trapezoidal rule.

3. Benchmark test

In this section, we outline our approach for comparing the numerical efficiency of the time- and frequency-domain methods for calculating acoustic pressure spectra.

$$\vec{x}_0(t) = \begin{pmatrix} x_0(t) \\ y_0(t) \\ z_0(t) \end{pmatrix} = \begin{pmatrix} a_0 + a_1t + a_2t^2 + a_3t^3 \\ b_0 + b_1t + b_2t^2 + b_3t^3 \\ c_0 + c_1t + c_2t^2 + c_3t^3 \end{pmatrix} \quad (11)$$

We consider two test cases. First, we examine simple, straight-line paths, for which the parameters a_2, a_3, b_2, b_3, c_2 and c_3 of equation (11) are zero. Second, we examine polynomial paths, which are defined by equation (11). In both cases, we examine a single frequency source signal with an amplitude of Q and a frequency of f_0 (see equation (6)). The parameters are chosen randomly within following intervals: $b_0, c_0 \in \left[-\frac{L}{20}, \frac{L}{20}\right]$ m, $a_1 \in \left[\frac{1}{5}\frac{L}{T}, \frac{L}{T}\right]$ m/s, $b_1, c_1 \in \left[-\frac{L}{20T}, \frac{L}{20T}\right]$ m/s, $a_2, b_2, c_2 \in \left[-\frac{L}{20T^2}, \frac{L}{20T^2}\right]$ m/s^2, $a_3, b_3, c_3 \in \left[-\frac{L}{20T^3}, \frac{L}{20T^3}\right]$ m/s^3, $f_0 \in [100, 500]$ Hz and $Q \in [0.5, 1.5]$ kg/s. Here, L is the characteristic length, set to 100 meters, and T is the observation duration, set to 2 seconds. The source's position along the x-axis at $t = 0$ is set at $a_0 = 0$m.

For the benchmark test, 1000 of such paths are selected and the corresponding pressure spectra $\hat{p}_F(f)$ (see equation (10)) and $\hat{p}_T$ (see equation (7)) are calculated. To ensure a fair comparison of the two methods, we calculate both their results using the same number of points N_s. The time t runs from $-\frac{T}{2}$ to $\frac{T}{2}$. For the time signal $p_T(t)$ (see equation (5)) we choose a sampling frequency f_S of 2 kHz. This results in $N_S = f_ST + 1 = 4001$ calculation points per path. In the frequency domain, we calculate the spectra from $f = 0$ Hz to $f = f_S/2$, on $N_S = 4001$ equally spaced points.

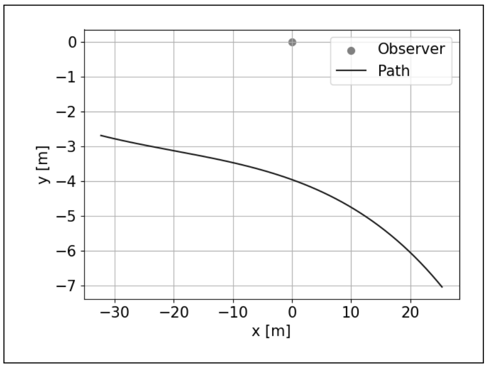

Figure 3: One of the polynomial paths and the observer position. The quasi velocity parameter in the x direction a_1 was chosen randomly at 28.83 m/s.

Finally, to ensure that both methods produce the same result, we examine the mean normalized difference between the two spectra, ϵ:

$$\epsilon = \mathrm{mean}_f\left(\frac{|\hat{p}_F(f) - \hat{p}_T(f)|}{\max_f|\hat{p}_F(f)|}\right) \quad (12)$$

4. Results

In this section, we present the results of the benchmark test comparing of the numerical efficiency of the

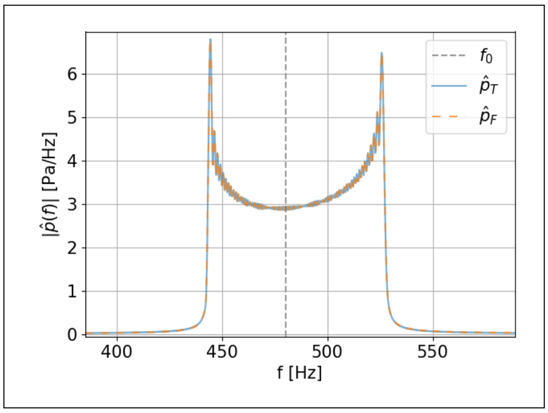

Figure 4: The received pressure spectrum, calculated using both the time domain method $\hat{p}_T$ (see equation (7)) and the frequency domain method $\hat{p}_F$ (see equation (10)), corresponding to the path shown in figure 3. The single frequency source frequency f_0 was randomly selected as 480.25 Hz.

methods in the time and frequency domains.
Our approach is described in section 3. Both straight-line and polynomial paths are considered. One such polynomial path is shown in figure 3. Figure 4 shows its corresponding received spectrum, which has an M-shaped curve with two peaks at the Doppler frequencies.

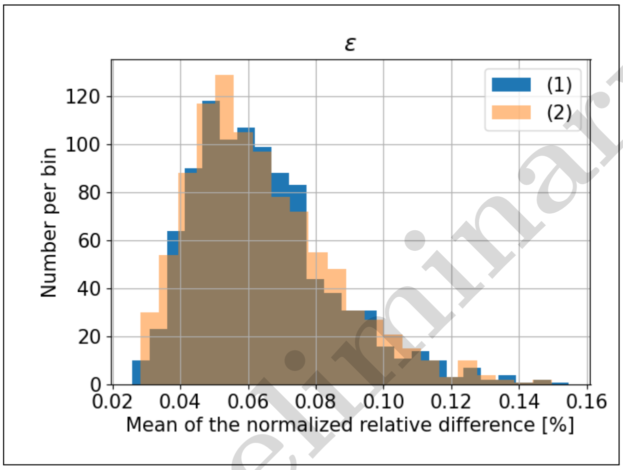

Figure 5: The histogram of the mean normalized difference ϵ (see equation (12)) between $\hat{p}_F(f)$ (see equation (10)) and $\hat{p}_T(f)$ (see equation (7)) for 1000 paths on N_S=4001 sampling points. Two cases are considered: Straight-line paths (case 1) and polynomial paths (case 2). See section 3.

First, we verify that the same results are achieved when using both the methods in the frequency and time domains. Figure 5 shows the histograms of the mean normalized difference, ϵ (see equation (12)), between both results for both cases. The distributions are slanted. However, the value of ϵ stays under 0.2 %.
Figure 6 shows the histograms of the calculation time for the pressure time signal, p_T (see equation (5)).

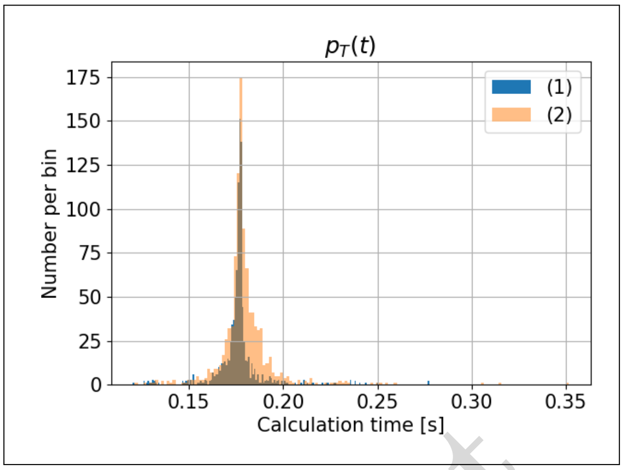

Figure 6: The histogram for the calculation time of $p_T(t)$ (see equation (5)) for 1000 paths on N_S=4001 sampling points. Two cases are considered: Straight-line paths (case 1) and polynomial paths (case 2). See section 3.

Both distributions have a peak at approximately 0.18 seconds.

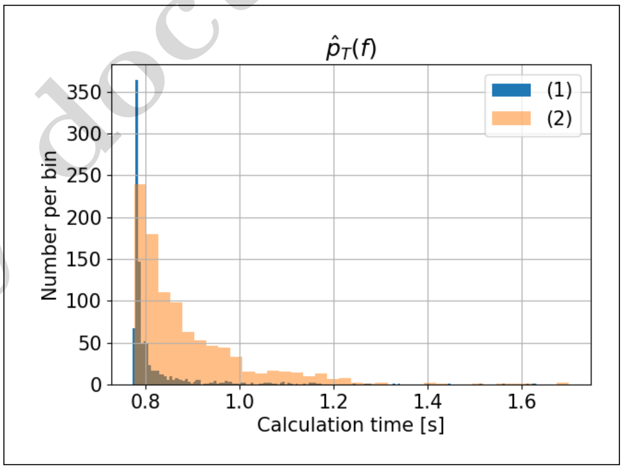

Figure 7: The histogram for the calculation time of $\hat{p}_T(f)$ (see equation (7)) for 1000 paths on N_S=4001 sampling points. Two cases are considered: Straight-line paths (case 1) and polynomial paths (case 2). See section 3.

Figure 7 shows the histograms of the calculation time for the Fourier transformation, $\hat{p}_T$ (see equation (7)), of the pressure time signal, p_T (see equation (5)), for both cases. Both distributions are heavily slanted, peaking at approximately 0.8 seconds. In most of the cases, the calculation time takes less than 1.2 seconds. Overall, the *time domain method* takes approximately one second to calculate.
Figure 8 shows the histograms of the calculation time of the frequency domain method, $\hat{p}_F$ (see equation (10)), for both cases. Similar to the Fourier Transformation calculation time, $\hat{p}_T$ (see figure 8), both diagrams are heavily slanted, peaking at approximately 0.9 seconds. In most cases, the calculation takes less than 1.4 seconds.

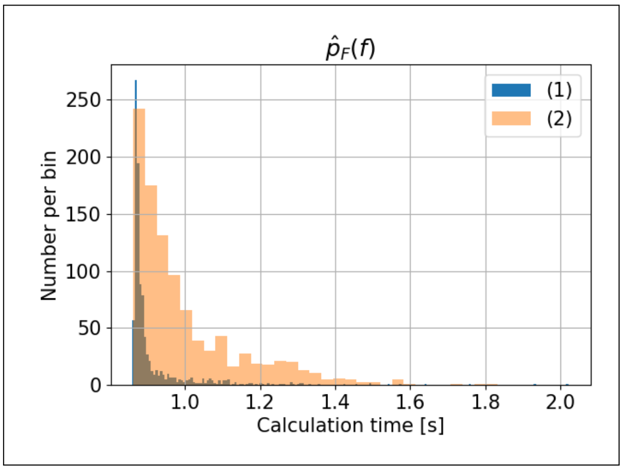

Figure 8: The histogram for the calculation time of $\hat{p}_F(f)$ (see equation (10)) for 1000 paths on N_S=4001 sampling points. Two cases are considered: Straight-line paths (case 1) and polynomial paths (case 2). See section 3.

Generally, calculations involving polynomial paths take longer than those involving straight-line paths (see figures 6 to 8).

5. Application

In this section, we examine the suitability of the frequency domain method for solving inverse problems involving rotating sources. These methods can be used to determine the location of noise sources on wind turbines or airplane rotors, for example. Essentially, inverse methods use measured acoustic signals and try to recreate them via a physical model of the event. The model's parameters are adjusted until the closest match between the measured and modeled signals is found.[6]

To evaluate if the measured and modeled signals are close, inverse methods use objective functions. In this section, we will propose such an objective function. Here, we consider a monopole rotating on a circular path, where all the parameters of the problem are assumed to be known, except the radius r_0 of the path. Then, we will evaluate our proposed objective function for various estimates $\tilde{r}_0$ of the radius r_0.

The circular path $\vec{x}_0(t)$ is given by:

$$\vec{x}_0(t) = \begin{pmatrix} \tilde{x}_0 + r_0 \cos(2\pi f_c t + \varphi_0) \\ \tilde{y}_0 + r_0 \sin(2\pi f_c t + \varphi_0) \\ 0 \end{pmatrix}, \qquad (13)$$

with $(\tilde{x}_0, \tilde{y}_0, 0)$ the center of the circular path, set at $(3, 3, 0)$ m, r_0 the radius of the path, set at 1 m, f_c the rotation frequency, set at 15 Hz, and φ_0 the phase offset, set at 0 rad. We also assume a single frequency source signal with an amplitude Q, set at 1 kg/s, and a frequency f_0, set at 300 Hz (see equation (6)).

For the preliminary examination of the objective function, we will use synthesized signals instead of actual, measured ones. We synthesize the measured signals $\hat{p}_1$ and $\hat{p}_2$ for two observers, positioned at $(x_1, y_1, z_1) = (1.5, 0, 1.5)$ m and $(x_2, y_2, z_2) = (3, 0, 1.5)$ m. Here, we use the frequency domain method (see equation (10)), whereby we evaluate the integrand at an equidistant grid corresponding to a sampling frequency f_S of 4 kHz and a observation duration T of 1 second.

For the objective function, the energy spectral density can be used [6]. Suppose two microphones, 1 and 2, measure an event. Then, the energy spectral density between the two received spectra, $\hat{p}_1$ and $\hat{p}_2$, is given by [6, chapter 2.3]:

$$C_{12}(f) = \frac{1}{2}\hat{p}_1(f)\hat{p}_2^*(f) \qquad (14)$$

whereby $*$ denotes the complex conjugate. We compare the energy spectral density of the measured signals $C_{12}(f)$, with that of the modeled signals $C_{12}^{\tilde{r}_0}$, which is also calculated using the frequency domain method (see equation (10)). Here, $\tilde{r}_0$ represents the estimate of the radius r_0. We consider the quadratic distance between $C_{12}(f)$ and $C_{12}^{\tilde{r}_0}$ [6, chapter 2.3].

To reduce the quadratic distance to a single scalar, we integrate over the frequency axis. Finally, we normalize the resulting value by dividing it by the integral of $C_{12}(f)$ over the frequency axis:

$$\mathcal{O}_\mathcal{D}(\tilde{r}_0) = \frac{\int_0^{f_S/2} ||C_{12}(f) - C_{12}^{\tilde{r}_0}(f)||^2 df}{\int_0^{f_S/2} ||C_{12}(f)||^2 df} \qquad (15)$$

$\mathcal{O}_D(\tilde{r}_0)$ is our objective function. Given the estimate $\tilde{r}_0$ it provides a scalar value that indicates how close the measured and modeled signals are.

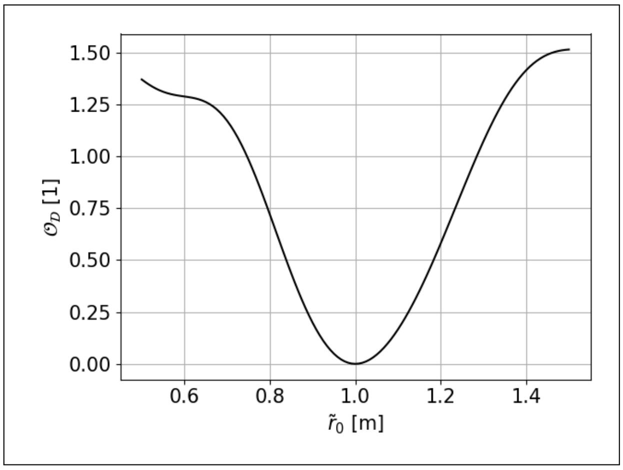

Figure 9: The objective function $\mathcal{O}_\mathcal{D}$ (see equation (15)) in function of the estimate $\tilde{r}_0$ of the radius r_0 (see equation (13)). The other parameters of the model (f_c, φ_0 etc.) are assumed to be known.

The result for the objective function $\mathcal{O}_\mathcal{D}$ in function of the estimate $\tilde{r}_0$ of the radius r_0 is shown in figure 9. A smooth curve is observed. The minimum of the objective function is found at the same radius r_0 as used for the synthetic measured signals.

6. Discussion

The results in Section 4 show that calculating the acoustic pressure spectrum using the frequency domain method is slightly quicker than using the *time domain method*. This is because the emission distance R only needs to be calculated numerically at the two integration limits when using the frequency domain method. However, the Fourier transformation in the time domain method $\hat{p}_T$ (see equation (7)), could be calculated using a fast Fourier transformation (FFT), which would reduce the calculation time. The expression for the received pressure spectrum in the frequency domain method $\hat{p}_F$, is not a standard Fourier transformation, due to the term in the exponent (see equation (10)). Therefore, an FFT generally cannot be used. Nevertheless, Zhang et al. [5] claim that the FFT can be used with the frequency domain method in certain cases, such as with rotating sources. An advantage of the frequency domain method is that if the received spectrum $\hat{p}$ only needs to be calculated for a small number of frequencies, the frequency domain method may be more efficient than a combination of calculating the time signal p_T and using an FFT to calculate the spectrum $\hat{p}_T$.

As shown in section 4, the same result can be achieved with both the frequency and time domain methods. The difference is negligible (see figure 5). This holds true even for more complicated paths, such as polynomial paths.

Section 5 shows promising results for the suitability of the frequency domain method for inverse problems involving rotating monopoles. The proposed objective function is smooth with respect to the estimation $\tilde{r}_0$ of the radius r_0 of the circular path (see figure 9). We could now apply this objective function to an inverse problem solver, which should be able to find the right radius r_0 in an effective manner.

This should stem from the nature of the received acoustic pressure spectrum of a rotating, single frequency, monopole source. These spectra consist nearly of lines, with a peak at the source signal's frequency f_0 , as well as other peaks before and after it. These lines are regularly spaced at with a distance corresponding to the rotation frequency f_c. Changing the radius r_0 of the circular path, only changes the amplitude of these lines and not their positions. The proposed objective function behaves much less smoothly then with respect to the rotation frequency f_c (this result is not shown here). This is precisely because changing f_c shifts the line positions.

As a next step, we propose studying how the objective function reacts to the changes in the other parameters of the model. For example we can study how it reacts to changes in the phase offset φ_0 of the monopole position (see equation (13)). Additionally, stochastic source signals, instead of single frequency sources, should be studied. So far, we have only considered two microphones. The next step is to consider entire microphone arrays. In the future, we would also like to apply this method to actual measurements.

7. Conclusion

This paper reviews the theory behind the time- and frequency-domain methods used to calculate the received acoustic pressure spectrum coming from a moving monopole source. The theory has been implemented, and the numerical efficiency of the methods in both domains has been compared. As demonstrated, the two methods produce the similar results with negligible differences. The frequency-domain method requires slightly less calculation time than the time-domain method. These results hold true for arbitrary paths. Advantages of the frequency domain method have been discussed. Finally, we present promising results for the suitability of the frequency-domain method to inverse problems concerning rotating sources.

8. Acknowledgments

This work was conducted as part of the "LION II: Localization and identification of moving noise sources" research project. We would like to thank the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for funding it - Project number 422372636. Our thanks also go to our colleagues at the DLR (German Aerospace Center), EMPA (Swiss Federal Laboratories for Materials Science and Technology) and ÖAW (Austrian Academy of Sciences) for our regular and very helpful discussions.

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