COMPARISON OF BENDING WAVE SPEED OF VISCOELASTIC PANELS USING TWO APPROACHES

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Abstract

This work employs a laser Doppler vibrometer (LDV) as a sensing modality for experimental measurements of bending wave properties in viscoelastic panels. A recently developed transfer function method [Xiang, N. *et al.* FA2025 Proc.] is applied to characterize these properties, with a focus on experimentally determining the bending wave propagation coefficient; a key quantity from which other bending wave properties can be derived. The LDV measures the bending velocities at two surface points on the panel, enabling the determination of the propagation coefficient as a primary parameter. From this, other properties such as the bending phase speed, Young's modulus, and the loss factor are obtained. Two approaches for determining the phase speed are examined: one based on the propagation coefficient and the other that calculates the phase spectrum of the transfer function. Following a brief introduction of the theoretical foundation, this paper discusses validation efforts for the experimentally measured bending phase speed using both approaches, with the goal of deriving additional parameters from propagation coefficient measurements.

Keywords: *Bending wave properties, bending wave speed, laser Doppler vibrometry, viscoelastic panels, transfer function method*

1. Introduction

Previous work [1] has reported preliminary investigations on experimental characterization of bending wave properties of viscoelastic panels. The experimental characterization effort uses a laser Doppler vibrometer (LDV) to investigate the transfer function method. This paper further develops the transfer function method in focusing on the bending wave propagation coefficient from a previously reported work by Xiang et al. [1]. The laser beam of the LDV perpendicularly senses bending velocities at two surface points, radially outwards from a point source that excites bending waves on a viscoelastic panel under test with a large enough size. This experimental characterization method relies on determining the transfer function of the bending velocities experimentally measured at the two surface points as the primary step. The transfer function is defined between the two responding surface points at known distances from the bending source. It becomes a key function of this dynamic method to analyze all the properties of the bending wave. These properties are derived from the propagation coefficient (wavenumber) of the bending wave, including the bending phase speed, Young's modulus, and the loss factor. The current paper reports on a further study of the experimental characterization to validate the approach centered around the propagation coefficient of the bending wave. This paper compares the phase speed measured from the system-theoretic approach using the phase function of the transfer function with that of resolving the phase speed from the propagation coefficient (the wave-propagation approach). The comparison effort aims to validate the transfer function method for characterization of the bending wave properties of viscoelastic panels.

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2. Bending Wave Theory

The propagation of the bending wave is governed by a fourth-order partial differential equation [2]

$$\nabla^4 \underline{V}_{\mathrm{b}} - \underline{\gamma}_{\mathrm{b}}^4 \underline{V}_{\mathrm{b}} = 0, \tag{1}$$

where a complex-valued propagation coefficient $\underline{\gamma}_{\mathrm{b}}$ is expressed [3] as

$$\underline{\gamma}_{\mathrm{b}} = \sqrt[4]{\frac{\omega^2 m''(1-\mu^2)}{\underline{E}}} = \sqrt[4]{\frac{\omega^2 m''(1-\mu^2)}{E_{\mathrm{R}}(1+\mathrm{j}\,\eta)}}. \tag{2}$$

with ω being angular frequency, m'' is specific mass density of the panel under test, a constant includes the panel density and other factors for simplicity, μ is *Passion*'s ratio. *Young*'s modulus $\underline{E} = E_{\mathrm{R}} + \mathrm{j}\, E_{\mathrm{I}}$ is complex-valued and $\eta = E_{\mathrm{I}}/E_{\mathrm{R}}$ is defined as the loss factor. Both parameters are of great significance in the building-acoustic engineering.
When a viscoelastic, isotropic panel features certain weak damping the propagation coefficient (wavenumber) $\underline{\gamma}_{\mathrm{b}}$ becomes complex-valued

$$\underline{\gamma}_{\mathrm{b}} = \alpha_{\mathrm{b}} + \mathrm{j}\, \beta_{\mathrm{b}}, \tag{3}$$

where α_{b}, β_{b} are the damping coefficient and the phase coefficient of the bending waves, respectively.

3. Bending Wave Transfer Function

The bending wave equation can be further approximated [3,4] in terms of bending velocity $\underline{V}_{\mathrm{b}}$ under condition $|\underline{\gamma}_{\mathrm{b}}|\, r \gg 1$ as

$$\underline{V}_{\mathrm{b}}(-\underline{\gamma}_{\mathrm{b}}\, r) \approx \mathrm{j}\sqrt{\frac{2}{\pi\, \underline{\gamma}_{\mathrm{b}}\, r}}\, \mathrm{e}^{-\underline{\gamma}_{\mathrm{b}}\, r}, \tag{4}$$

for a forward propagating component. Building upon the approximation as in Eq. (4), this paper further investigates the experimental method at two surface points r_1, r_2 with $r_1 < r_2$, radially outwards from a point source exciting the bending waves on the viscoelastic, isotropic panel under test.

3.1. System-theoretic approach

In system-theoretic approach, the transfer function can be expressed in terms of its magnitude and phase spectra

$$\underline{H}_{\mathrm{b}} = \frac{\underline{V}(-\underline{\gamma}_{\mathrm{b}}\, r_1)}{\underline{V}(-\underline{\gamma}_{\mathrm{b}}\, r_2)} = |\underline{H}_{\mathrm{b}}|\, \mathrm{e}^{-\mathrm{j}\phi(\omega)}. \tag{5}$$

The phase delay time derived from the phase spectrum $\phi(\omega)$ given the separation of two surface points leads to the phase speed of the bending wave [1] by

$$c''_{\mathrm{b}} = \frac{\omega}{\phi(\omega)}(r_2 - r_1). \tag{6}$$

3.2. Wave-propagation approach

In wave-propagation approach, the transfer function $\underline{H}_{\mathrm{b}}$ can also be determined using Eq. (4) for $|\underline{\gamma}_{\mathrm{b}}|\, r \gg 1$

$$\underline{H}_{\mathrm{b}} \approx \sqrt{\frac{r_2}{r_1}}\, \mathrm{e}^{-\underline{\gamma}_{\mathrm{b}}\,(r_1 - r_2)}. \tag{7}$$

The transfer function in terms of bending wave propagation as in Eq. (7) enables solving for the propagation coefficient $\underline{\gamma}_{\mathrm{b}}$. Upon solution of $\underline{\gamma}_{\mathrm{b}}$, the phase speed is determined by

$$\underline{c}_{\mathrm{b}} = c'_{\mathrm{b}} - \mathrm{j}\, c''_{\mathrm{b}} = \frac{\omega}{\underline{\gamma}_{\mathrm{b}}}, \tag{8}$$

where c'_{b} corresponds to damping and c''_{b} is the phase speed of the bending wave. Eq. (2) also leads to Young's modulus and the loss factor by

$$\underline{E} = E_{\mathrm{R}}(1 + \mathrm{j}\, \eta) = \frac{\omega^2}{\underline{\gamma}_{\mathrm{b}}^4}\, m''(1 - \mu^2). \tag{9}$$

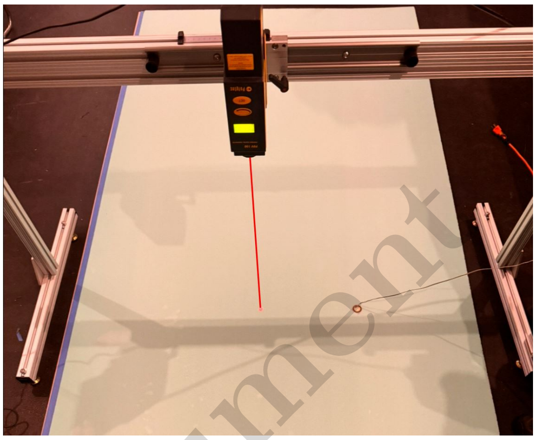

Figure 1: Laser Doppler vibrometer-based measurement setup of a viscoelastic, isotropic panel under test.

4. Preliminary Experimental Results

Figure 1 shows a photograph of a measurement setup using a laser Doppler vibrometer (LDV). A viscoelastic panel [1] of 6.4 mm thick with an areal size of 1.22 m × 1.83 m is used as a sample under test. The panel with a mass per unit area m'' of 184 [g/m^2] is elastically supported. A light-weight piezoeletric thin disk is used as a point source to excite bending waves. When attached at an appropriate location, the laser beam of the LDV is perpendicularly shinning onto the panel surface. The outputs of the LDV are proportional to the bending wave velocities. In this example, the bending velocities at two points ($r_1 = 23$ cm, $r_2 = 25$ cm), radially outwards from the piezoelectric exciter are processed to build the transfer function. The experimentally measured phase speed in Eq. (6) helps estimating the far-field condition $|\underline{\gamma}_{\mathrm{b}}|\, r \gg 1$ which is largely fulfilled over the frequency range of interest. Figure 2 shows one pair of experimentally measured impulse responses between the input of the bending wave exciter and the outputs of the LDV at the two radial locations.

Figure 2 shows that the direct arrival of the bending wave at the position for $r_2 = 25$ cm exhibits a lower amplitude than at $r_1 = 23$ cm, demonstrating a damping behavior of the bending wave by the isotropic viscoelastic panel. A time window is carefully positioned as shown in Fig 2 (a), which can eliminate

[1] *Kingspan* **GreenGuard**® Type IV 25 PSI XPS Insulation Board.

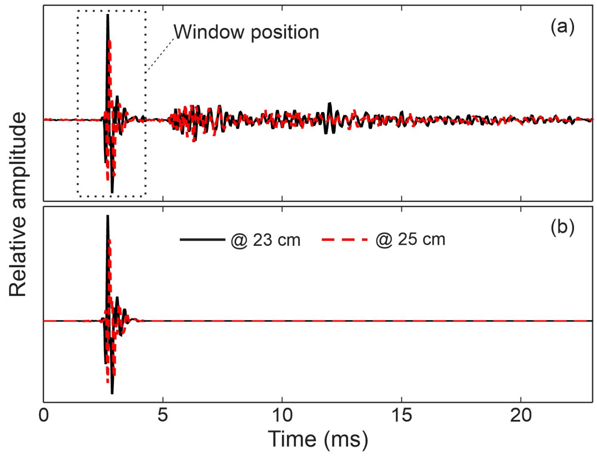

Figure 2: Segments of impulse responses measured at two points 23 cm and 25 cm from the source. (a) Raw impulses responses. (b) Windowed impulse responses.

unwanted reflections inevitably from the boundaries of the panel [see Fig 2 (b)]. The two windowed impulses will be zero-padded to take a sufficient temporal length for a desired frequency resolution.
The fast Fourier-transform of the windowed impulse responses followed by cross-spectrum calculation yields the transfer function as stated in Eq. (7). The magnitude and phase spectra of the bending velocity transfer function are illustrated in Fig. 3.

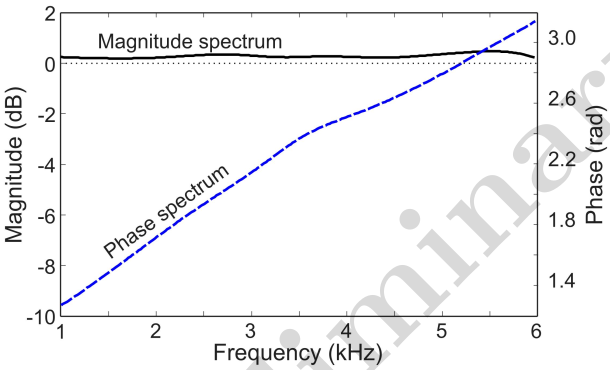

Figure 3: Magnitude and phase spectra of the transfer function of the bending waves between two surface points (r_1, r_2) experimentally measured on the viscoelastic panel under test.

5. Phase Speed Comparison

The complex-valued propagation coefficient $\underline{\gamma}_{\text{b}}$ is determined from the transfer function experimentally measured as in Eq. (7). This leads to the phase speed c_{b}'' of the bending wave using Eq. (8). In addition, the phase speed can also be determined using the system-theoretic approach in Eq. (6) from the phase spectrum $\phi(\omega)$ of the transfer function as reported previously by Xiang et al. [1], this study compares the phase speeds using the two different approaches to validate the essential relations of the propagation coefficient to the transfer function derived within the scope of the current work.

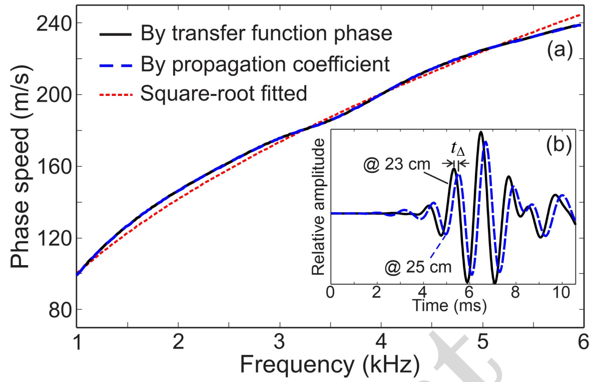

Figure 4: Comparison of phase speeds using two approaches. (a) From the propagation coefficient, and from the phase function of the transfer function. (b) Band-pass filtered direct impulses for validating the measured phase speed by estimating the time difference t_Δ.

Figure 4 compares the phase speeds experimentally determined in two distinctly different ways (one is system-theoretical, the other is wave-propagation). Highly agreeable phase speeds both from the phase function of the transfer function [Eq. (6)] and from the complex-valued propagation coefficient [Eq. (8)] evidently validate the derivations of the propagation coefficients from the transfer function.
Infinite-impulse response (IIR) filtering of the bending wave impulse responses over narrow band-widths (1 kHz - 1.5 kHz, 3.5 - 4 kHz and 5.5 - 6 kHz) can be used to estimate the time differences between two impulse responses. The time difference t_Δ between the narrow band-pass filtered impulse responses is used to estimate the phase speeds for different frequency ranges that serve another way to validate the phase speeds experimentally measured. The separation of two radially outwards points amounts to 2 cm.
Equation (9) effectively supports estimation of Young's modulus. Figure 5 illustrates the real and imaginary parts of Young's modulus using this setup.

6. Summary

This paper reports on further investigations of the experimental method to determine the bending phase speed through the complex-valued propagation coefficient. It is a key quantity for characterizing the bending wave properties of viscoelastic isotropic panels, including the bending wave speed, Young's modulus, and the loss factor. The laser Doppler vibrometer is applied in the transfer function method of characterizing viscoelastic panels. This work tries different ways to validate the experimentally measured phase speed on which the other bending parameters are also based. The current effort is being made to experimentally characterize the bending wave properties including Young's modulus and the loss factor.

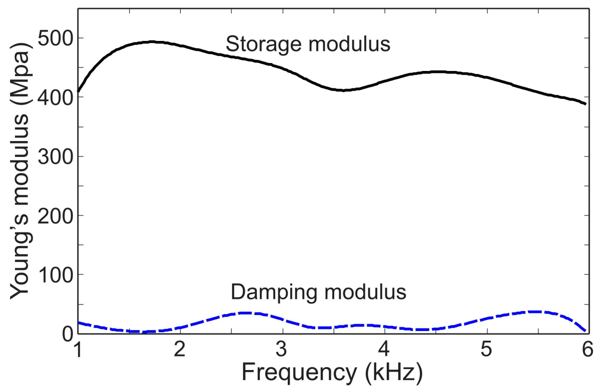

Figure 5: Comparison of phase speeds obtained using two approaches. (a) Determination from the propagation coefficient, and from the phase function of the transfer function. (b) Band-pass filtered direct impulses for validating the measured phase speed by estimating the time difference t_{Δ}.

References

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