MODAL TRANSFER MATRIX MODELING OF MULTILAYERED PLATES UNDER POINT FORCE EXCITATION IN BUILDING ACOUSTICS

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Abstract

For structure-borne sound problems, the driving-point mobility at the excitation position is a key quantity for estimating the power introduced into a structure by an excitation source. Previous studies have shown that approximating the driving-point mobility by a constant characteristic mobility can lead to substantial deviations, particularly at low frequencies, near boundaries, and for layered building components. For such components, the Transfer Matrix Method is a well-established approach for modeling sound transmission. Modal extensions further allow the response of rectangular plates with simply supported boundary conditions to be considered. In the present work, the modal Transfer Matrix Method (mTMM) is applied to evaluate the driving-point mobility of a multilayered, simply supported solid plate subjected to a point force excitation on the upper layer, representing a structure-borne excitation. For the investigated reference cases, the proposed mTMM approach shows good agreement with conventional finite element simulations and alternative analytical models. At the same time, it retains the computational efficiency and low modeling effort previously demonstrated for the mTMM compared with full finite element models. However, the restriction to simply supported plates limits the applicability of the method to practical building structures. To address this limitation, a Ritz-type extension based on a truncated sine basis is proposed to include rotational boundary stiffness within the modal Transfer Matrix framework. Initial results indicate the feasibility of the approach and provide a proof of concept for further development.

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1. Introduction

Housing shortages, urbanisation, and increasing residential density call for new planning strategies for densification, repurposing, and renovation [1], [2]. This is likely to increase the relevance of neighborhood noise and structure-borne sound in buildings. Alongside impact sound, building services equipment represents a widespread source of structure-borne sound [3].
In contrast to practical methods against impact sound, protection against installation noise must take into account not only the constructional conditions, but also the properties of the installed noise source [4]. Conceptually, both the source quantities themselves and the physical coupling between the installation and the excited building component are relevant [5].
Those coupling conditions are described by means of the driving point mobility, which significantly influences the installed structure-borne sound power from building services equipment. The latter sound power is generally given by EN 15657 [6] as

$$L_{W inst} \approx 10 \lg \left(\frac{\mathrm{Re}\left[Y_{R,eq}\right]}{Y_0 \cdot \left(1 + \frac{|Y_{R,eq}|^2}{|Y_{S,eq}|^2}\right)} \right) + L_{Fb,eq} \ \ [\mathrm{dB}], \tag{1}$$

and discussed in the broader context, among others, by Schöpfer et al. [5], [7]. The equivalent receiver mobility $Y_{R,eq}$ and the equivalent source mobility $Y_{S,eq}$ including a reference of $Y_0 = 1\,\mathrm{m/(Ns)}$ are part of the calculation. Further, the equivalent, blocked forces $L_{Fb,eq}$ of the source are needed and typically measured [6]. For homogeneous, solid and single-layered receiving plates the driving-point mobility $Y_{R,eq}$ is given for practical application by the standard EN 12354-5 Annex F with the characteristic mobility

$$Y_{char} = \frac{1}{8\sqrt{m' \cdot B}} \ \ [\mathrm{m\,s^{-1}\,N^{-1}}] \tag{2}$$

as a constant approximation [4]. Alongside the areal mass m' the bending stiffness B is needed for the calculation. The current, practical limitations for solid plates are therefore set on homogeneous, single-layered and infinite plate structures. However, in practical situations, especially for layered plates such as floating floors and in the vicinity of boundaries, current standards and the available literature do not provide adequate solutions at low frequencies. Previous literature [8], [9], [10], [11] worked on those issues with respect to finite dimensions and near the boundaries. Therefore, the dynamic behavior of layered structures is considered. Established finite element simulations provide high accuracy, but usually require considerable modeling effort. In the context of practical building acoustics, efficient alternatives with sufficient accuracy are needed. Hence, analytical expressions and modeling approaches for calculating the required quantities with comparatively low modeling effort are evaluated. Consequently, the present paper addresses the dynamics of layered, solid structures. The focus is placed on the well-known Transfer Matrix Method (TMM), including several modifications to meet the requirements for usage in the given practical context. The research aim is placed on defining an analytical model able to calculate the driving point mobility of multilayered plates with finite dimensions and arbitrary rotational boundary stiffness. The frequency range is limited up to 1000 Hz, since at higher frequencies characteristic values are sufficient [8].

In the course of the paper, the mobility Y is visualized in logarithmic scale and as a level by

$$10\ \lg\left(\frac{|Y|}{Y_0}\right)\ [\mathrm{dB}]\ \text{with}\ Y_0 = 1\,\mathrm{m\,N^{-1}s^{-1}}. \tag{3}$$

2. Transfer Matrix Method and Modal Extension

In the context of the research aim three key points need to be satisfied by the chosen method. First, a single point source of structureborne noise is needed. Second, finite dimensions with the corresponding support condition should be included. And third, the output parameter has to be the driving-point mobility to be consistent with the practical conditions of the coupling term of equation 1.

The TMM is an established and well researched method for calculating the sound propagation in layered materials based on analytical expressions. Developed for the calculation of wave propagation in layered media, it is often used for sound transmission between source and receiving rooms [12], [13]. While multiple methods investigate finite dimensions by windowing the radiation of an infinite plate [14], Vastiau et al. [13] introduce a TMM alteration by using a mobility based expression with modal filtering to include finite dimensions (mTMM).

The general TMM describes the dependencies between stresses and velocities on the top and the bottom surfaces of the considered layer by means of a transfer matrix $\mathbf{T}$ with the entries T_{ij} [13]. The detailed formulae describing T_{ij} are found for example in Geebelen [12]. Vastiau et al. [13] rework the equation system initially based on the transfer matrix $\mathbf{T}$ to derive a transfer function matrix $\mathbf{H}$. This transfer function matrix enables to couple the force F on the top surface to the velocity v on the bottom surface by using only the known entries T_{ij}. For a linear elastic media, the matrices are of $[4 \times 4]$ dimension and can be expressed as a mobility ($Y = v/F$ [15]) by the means of certain entries T_{ij}, resulting in

$$\mathbf{Y}_\infty(k_r, \omega) = \frac{T_{41}}{T_{32}T_{41} - T_{31}T_{42}}. \tag{4}$$

Hence, $\mathbf{Y}_\infty(k_r, \omega)$ represents the mobility of an infinite, layered plate, and is a function over the radial bending wave number k_r and the angular frequency ω [13]. By weighting this infinite response with the help of a modal extension for simple support condition, Vastiau et al. [13] define

$$\mathbf{Y}_m(x_f, y_f, k_m, k_n) = \frac{4}{L_x L_y}\Phi(x_f, y_f)\hat{\mathbf{H}}(k_m, k_n, \omega) \tag{5}$$

in general to derive the finite transfer mobility. $\Phi(x_f, y_f)$ represents the modal shapes of simple supported boundary conditions, L_x, L_y are the finite dimensions, x_f, y_f the input position, and $\hat{\mathbf{H}}(k_m, k_n, \omega)$ resembles the transfer function matrix in general. In this context, the transfer function matrix $\hat{\mathbf{H}}$ is set to be equal to the mobility $\mathbf{Y}_\infty(k_r, \omega)$, and evaluated at the specific modal wave numbers

$$k_m = \frac{m\pi}{L_x},\ \ k_n = \frac{n\pi}{L_y} \tag{6}$$

corresponding to the simply supported boundaries [13].

It should be added that different material classes can be implemented in the TMM namely elastic, poroelastic materials and fluids resulting in different sized transfer matrices [12]. Further, extensions to deal with air pressure loading on the outer surfaces are available [13]. In the context of the presented research only elastic material properties and negligence of the air pressure loading were implemented for practicability and efficiency.

3. Model Modification for Driving-Point Mobility and Rotational Stiffness at the Boundaries

While Vastiau et al. [13] use the transfer mobility from the force input to the receiver room sided surface to calculate the radiation into the receiving room, the present research reformulates the mobility expression to calculate the source sided driving-point mobility Y_R for the coupling term of equation 1. Following the derivation principle of the transfer function matrix, $\mathbf{Y}_{\infty,R}$ is now given to

$$\mathbf{Y}_{\infty,R}(k_r,\omega) = \frac{T_{21}T_{42} - T_{22}T_{41}}{T_{31}T_{42} - T_{32}T_{41}} \tag{7}$$

with the corresponding entries T_{ij} taken from Geebelen [12]. Similarly, this driving-point mobility of the infinite plate is transformed into the finite response by applying the modal extension (eq. 5) with $\hat{\mathbf{H}}(k_m, k_n, \omega) \equiv \mathbf{Y}_{\infty,R}(k_r, \omega)$. Hence, an analytical expression of the driving-point mobility of a simply supported, multilayered solid plate is given.

The simply supported boundary conditions are commonly used as idealized boundary conditions in theoretical derivations. Neves e Sousa and Gibbs [10] argue that such conditions provide a sufficient approximation, especially at low frequencies. However, Aoki et al. [8] suggest low rotational support stiffness after calibrating an analytical model with rotational boundary stiffness against measurements. Furthermore, asymmetric boundary conditions with different rotational stiffnesses, ranging from simply supported to clamped edges, are of practical relevance [16]. Therefore, methods for extending the mTMM towards varying rotational boundary stiffness are evaluated.

This paper proposes an initial methodology to address this limitation. The mTMM after Vastiau et al. [13] is, in its current form, limited to simply supported boundaries, since the modal extension (eq. 5) filters the infinite TMM solution by means of modally decoupled wavenumbers. Rotational stiffness and clamped boundary conditions, however, introduce modally coupled bending wavenumbers. Consequently, the response cannot be represented by assigning one wavenumber to exactly one corresponding infinite TMM value.

To keep the simplicity of the mTMM and to simultaneously include rotational stiffness, a sine-based Ritz approach is introduced. In general, the Ritz approach approximates the displacement field by a finite set of basis functions and an energy equilibrium formulation [17]. The proposed approach follows this general Ritz formulation and combines the modal dynamic stiffness D_{mTMM} of the layered plate, obtained from the mTMM, with an additional boundary stiffness K_{BC} derived from the sine-base Ritz formulation. Intuitively, the boundary rotation associated with the rotational restraints is approximated by a weighted sum of sine-basis functions. To enable the combination of both approaches, the discrete wave numbers of the simply supported mTMM with the corresponding sine-based form functions are used as Ritz base functions. Consequently, the generalized displacement amplitudes $q(\omega)$ are obtained from

$$q(\omega) = [K_{BC} + D_{mTMM}(\omega)]^{-1}\, F \tag{8}$$

with ω the angular frequency and F the generalized force vector. As a result, this proposed approach covers rotational restraints by including a line stiffness k_ϑ along the boundaries into the boundary stiffness matrix K_{BC}. Hence, support situations between the simply supported and clamped limit cases with $k_\vartheta = 0$ and $k_\vartheta = \infty$ are covered. Finally, the mobility is calculated from the velocity response and the applied unit force $F = 1.0\,\mathrm{N}$, with the velocity obtained as the time derivative of the displacement [15].

4. Results and Discussion

The following section introduces the results of the proposed model extensions. First, the model based on equation 7 resulting in the driving point mobility is discussed. Second, the sine-base extension is evaluated at two limit cases. Both model extensions are implemented in python and use a realistic plate construction with material and geometric parameters listed in table 1. Lastly, a brief discussion of computational efficiency is added to address the practicability of the analytical expressions over full FEM simulations.

Table 1: Geometric and material based parameters of the researched system (DIN 18560-2 [18] and Fraunhofer-IBP internal material values).

Base plate	**Parameter**
Dimension L_x, L_y	$4.96 \times 3.96\,\mathrm{m}$
Thickness t	$0.14\,\mathrm{m}$
Young's Modulus E	$27.5\,\mathrm{GPa}$
Poisson's Ratio ν	0.20
Loss Factor η	0.05
Density ρ	$2300\,\mathrm{kg/m^3}$
Insulation	**Parameter**
Thickness t	$0.025\,\mathrm{m}$
Young's Modulus E	$3.0\,\mathrm{MPa}$
Poisson's Ratio ν	0.20
Loss Factor η	0.05
Density ρ	$50\,\mathrm{kg/m^3}$
Dyn. stiffness s'	$40\,\mathrm{MN/m^3}$
Cement Screed	**Parameter**
Thickness t	$0.045\,\mathrm{m}$
Young's Modulus E	$30.0\,\mathrm{GPa}$
Poisson's Ratio ν	0.20
Loss Factor η	0.05
Density ρ	$2200\,\mathrm{kg/m^3}$

4.1. Results and Discussion of the Driving-Point Mobility

For model evaluation, the analytical results are compared with finite element simulations conducted in the commercially available software COMSOL Multiphysics. Different physic interfaces can be used to model multilayered structures. In this study, the "Layered Shell" and "Solid Mechanics" interfaces are considered. The "Layered Shell" interface represents the laminate by a surface geometry combined with an additional thickness description and is based on a layerwise shell formulation [19]. In contrast, the "Solid Mechanics" interface models the structure as a three-dimensional continuum and discretises the full volume [20]. The comparison therefore evaluates the analytical model against both a layered-shell representation and a more general three-dimensional solid-mechanics model.

At this stage, the mesh is generated automatically, while the maximum element size is controlled to provide a sufficiently fine discretisation over the investigated frequency range. As a practical criterion, approximately five to six elements per shortest bending wavelength are taken [21].

Using the material and geometric parameters listed in Table 1, simply supported, layered plates subjected to a dynamic point force at $(x, y) = (1.0\,\mathrm{m}, 1.0\,\mathrm{m})$ are investigated. The resulting driving-point mobilities from both FEM interfaces and from the mTMM simulation are plotted in Figure 1.

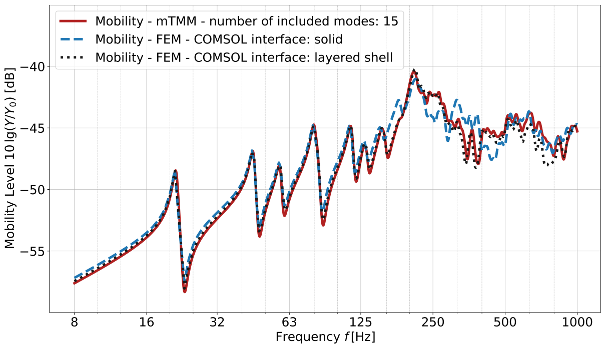

Figure 1: Driving-point mobilities – FEM / mTMM – simply supported – 3 layers – Position: $x = 1\,\mathrm{m}, y = 1\,\mathrm{m}$ – geometry and material: Table 1

The compared data show reasonable agreement, particularly at low frequencies. Towards higher frequencies, the results obtained with the "Solid Mechanics" interface visibly deviate from both the mTMM and the "Layered Shell" results. This deviation can be attributed to a combination of different modeling assumptions and different representations of the boundary conditions. While the transfer matrix formulation accounts for wave-based effects in the thickness direction, the finite modal extension describes the global response by a plate-type modal representation. The observed deviations occur in the transition from the modal low-frequency behavior to the apparent dynamic decoupling of the layered structure above approximately 125 Hz.

In addition, the boundary conditions are not represented identically in the models. In the mTMM and "Layered Shell" models, the applied support conditions act on every layer, whereas the "Solid Mechanics" model allows the more realistic configuration in which only the edges of the base plate are simply supported. Consequently, the mTMM shows closer agreement with the "Layered Shell" model than with the "Solid Mechanics" model. This indicates that the comparison is influenced not only by the underlying modelling theory, but also by the idealisation of the supports. Further investigations and comparisons with measured data are required to assess the feasibility of the approach more comprehensively. Nevertheless, within the investigated frequency range, the mTMM approach shows good agreement with the "Layered Shell" data.

With the proposed rearrangement of the mTMM, the driving-point mobility of the bare base plate can be compared with that of the layered construction. Figure 2 illustrates this comparison and additionally shows the characteristic mobilities of the screed and the base layer. The single-layered base plate is calculated by the use of a classical modal approach following the standard theory [15]. At low frequencies, pronounced modal behavior is observed for both systems. However, due to the additional floating floor system and the resulting differences of the areal masses the resonances shift to lower values. In a separate comparison, not shown here, the areal mass of the screed and insulation were added artificially to the base plate, resulting in close agreement of the low resonant frequencies giving expected behavior. While the mobility of the base plate oscillates around its characteristic mobility (after equ. 2), the mobility of the layered structure increases with frequency and eventually approaches the characteristic mobility of the screed. This behavior indicates the mass decoupling of the floating floor roughly above the visualized resonance frequency f_0 of the mass-spring-mass floor system following

$$f_0 = \frac{1}{2\pi}\sqrt{s'\left(\frac{1}{m'_{top}} + \frac{1}{m'_{bottom}}\right)} \tag{9}$$

with the dynamic stiffness s' and the areal masses m' of both solid plates [22]. Those results motivate further parametric studies to investigate the dependencies governing the increasing mobility at low frequencies, since this transition extends well into the frequency range relevant to building acoustics.

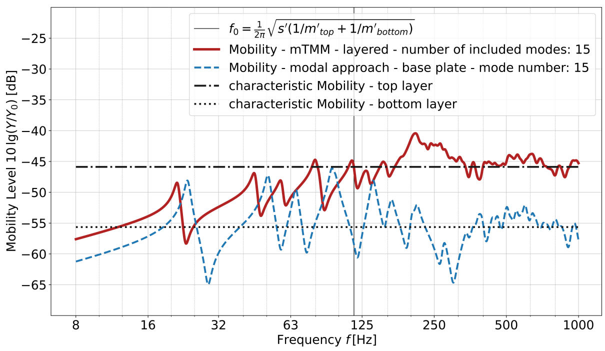

Figure 2: Driving-point mobilities – modal / mTMM – simply supported – one-layered base plate / three-layered plate (floating floor) – Position: $x = 1\,\mathrm{m}, y = 1\,\mathrm{m}$ – geometry and material: Table 1

4.2. Simple and Clamped Boundaries

For initial validation of the sine-based Ritz-approach the two limit cases $k_\vartheta = 0$ and $k_\vartheta = \infty$ are evaluated. Additionally, only one layer is modeled in the mTMM method to provide comparability with results from idealized simple and clamped boundary conditions calculated by a modal approach implementation. First, the limit case of a rotational stiffness $k_\vartheta = 0$ gives expected, simply supported results, stating the correct behavior by cancellation of K_{BC}. Second, a high rotational stiffness of $k_\vartheta = 10^{12}\,\mathrm{N}$ is chosen to approximate clamped support conditions. Figure 3 plots the results of the approximate clamped supports of the mTMM. Notably, the number of sine functions was determined through a manual and iterative process.
Based on a visual comparison, the proposed approach reproduces the main characteristics of the reference solution and motivates further investigations. Hence, detailed investigations on numerical convergence are necessary as the calculation of the preliminary results showed compensation effects between model theory and numerical errors regarding the number of included mode functions.

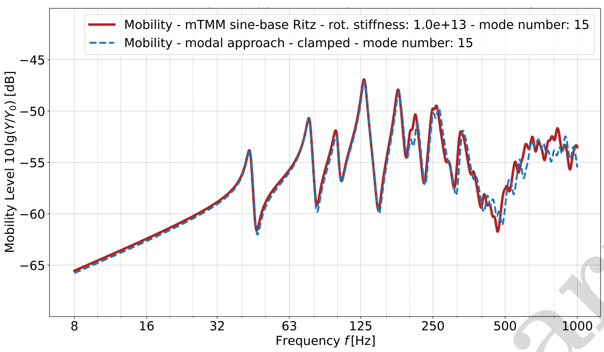

Figure 3: Driving-point mobilities – modal / mTMM incl. sine-base Ritz – clamped supported – one-layered base plate – Position: $x = 1\,\mathrm{m}, y = 1\,\mathrm{m}$ – geometry and material: Table 1

4.3. Computational Efforts

As discussed above, the motivation for using analytical models lies in their computational efficiency and reduced modeling effort. Although the latter is difficult to quantify systematically, the computational effort can be assessed by means of the required calculation time. Table 2 summarizes representative computational requirements for the most demanding mTMM configuration considered in this study, namely the layered floating floor system, and the computationally least demanding FEM configuration, namely the layered-shell model. This selection therefore provides a conservative comparison with respect to the analytical model. Neither implementation was specifically optimized for computational efficiency. The individual models were configured according to the respective state of the art to obtain sufficiently reliable results. The frequency range and frequency spacing were identical for both calculations. Consequently, the reported values should be interpreted as an indicative runtime comparison rather than as a systematic performance benchmark. Nevertheless, for the investigated configuration and implementation, the runtime of the analytical model is well below one minute demonstrating real time analysis of the investigated systems.

Table 2: Comparison of the computational efforts

FEM COMSOL	**layered shell**
CPU	Intel Core Ultra 9 2.50GHz
RAM	64GB
end-to-end runtime	2758 s
mTMM	**3 layers**
CPU	Intel Core i9-9900 3.10GHz
RAM	32GB
end-to-end runtime	9 s

5. Conclusion and Outlook

The present paper addresses the efficient calculation of the driving point mobility in the methodological framework of practical and normative applications. For this purpose, a suitable modeling approach is identified and adapted to the relevant boundary conditions, including finite plate dimensions, layered structures, and the evaluation of the driving point mobility caused by a single point force. In addition, an approach for including elastic rotational boundary stiffness is proposed based on a sine-base Ritz method. The discussion of the results demonstrates generally good agreement between the chosen reference solutions and gives therefore a proof of concept. A more detailed methodological discussion and further validation will be presented in future work.

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