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A STOCHASTIC MODAL ENERGY DECAY MODEL FOR THE ESTIMATION OF ABSORPTION

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Abstract

The measurement of Sabine's random incidence absorption coefficient as obtained from reverberation room measurements is well-known to suffer from poor inter-laboratory reproducibility. This issue is often attributed to an insufficiently diffuse and distinctly non-uniformly damped sound field within the laboratory, which is most pronounced when the absorbing test specimen is mounted. Characteristic multi-exponential energy decay curves and corresponding multimodal damping distributions are extensively reported already in the early literature on the topic. In recent years, multiple methods have been investigated to account for non-uniform damping, either through explicit or implicit numerical modeling or when inferring the reverberation time of the empty and occupied laboratory.

In this contribution we analyze the uniformity of damping using an energy-based stochastic modal damping model originally introduced by Kuttruff. Discrete damping distributions are computed for a rectangular reverberation room with non-uniform damping based on the closed form solution for rectangular rooms. Finally, we investigate the suitability of the damping distributions of the occupied and empty reverberation room for the estimation of Sabine's random incidence absorption coefficient.

Keywords: *MANDATORY, 3 to 5 keywords, separated by a comma, italic, not bold, no capitalized letters*

1. Introduction

The presence of non-homogeneous modal damping in rooms with concentrated absorbing material on a single surface – as is the case during the measurement of Sabine's absorption coefficient as per [1] – is a well-known phenomenon [2]. Already in the 1930s, Hunt et al. [3] showed that axial, tangential, and oblique modes in a rectangular room decay at different rates. In the case of concentration of absorption on a single surface, they found that modes corresponding to grazing incidence are significantly less damped than modes corresponding to non-grazing incidence. The relationship between the damping distribution of the sound field and the resulting multi-exponential energy curve was further investigated by Kuttruff [4] using the Laplace transform. Brüel suggested using the initial energy decay rate, rather than the standard T_{20} or T_{30} regression interval, to estimate equivalent reverberation-room absorption in sound power measurements. Jacobsen [5] however theoretically showed that the influence of normal and tangential modes at low frequencies may be incorrectly neglected. Balint et al. [6] suggested to identify the reverberation times using a multi-exponential energy decay model when estimating Sabine's absorption coefficient. They used the shortest reverberation time, corresponding to the highest damping, for the estimation. An informal inter-laboratory study, however, indicated large errors in the calculated absorption coefficients in one out of three laboratory rooms [7].

In this contribution, we investigate the suitability of the energy-weighted mean damping, i.e. the average damping of all modes constituting the sound field, instead of using only the highest damping for the estimation of Sabine's absorption coefficient. Sections 2 and 3 introduces a stochastic energy decay model inspired by Kuttruff [4] and its application to the estimation of Sabine's absorption coefficient. We present a simulation study and the corresponding results in Sections 4 and 5.

2. Modal Energy Decay

The energy decay of a single mode can be calculated from the first-order differential equation [5]

$$\frac{\mathrm{d}E_\ell(t)}{\mathrm{d}t} = -\delta_\ell E_\ell(t), \tag{1}$$

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where $E_\ell(t)$ is the energy of the mode ℓ at time t. The solution to the differential equation is the well-known exponential decay function

$$E_\ell(t) = E_{\ell,0} e^{-\delta_\ell t}. \tag{2}$$

Following Jacobsen [5], the damping factor δ_ℓ is calculated as the ratio of sound power absorbed at the room boundaries and the energy of the mode

$$\delta_\ell = \frac{\int_S \mathbf{I}_\ell \cdot \mathbf{n} \mathrm{d}S}{\int_V w_{\mathrm{pot},\ell} + w_{\mathrm{kin},\ell} \mathrm{d}V}, \tag{3}$$

where $\mathbf{I}_\ell \cdot \mathbf{n}$ is the normal component of the active intensity vector on the surface element $\mathrm{d}S$, and $w_{\mathrm{pot},\ell}$ and $w_{\mathrm{kin},\ell}$ are the potential and kinetic energy of the mode, respectively. For rectangular rooms with non-uniform impedance distributions as given by Nolan and Davy [8], Eq. (2) can be rewritten as

$$\delta_\ell = \frac{2c \int_k k\Lambda_\ell \int_S \Re\{\psi_\ell(\mathbf{x}, \mathbf{k}_\ell)\nabla\psi_\ell(\mathbf{x}, \mathbf{k}_\ell)\} \cdot \mathbf{n} \mathrm{d}S \mathrm{d}k}{\int_k \Lambda_\ell \left[\int_V |\psi_\ell(\mathbf{x}, \mathbf{k}_\ell)|^2 + k^2 |\nabla\psi_\ell(\mathbf{x}, \mathbf{k}_\ell)|^2 \mathrm{d}V\right] \mathrm{d}k}, \tag{4}$$

where $\mathbf{x}$ is a Cartesian coordinate vector, and c is the speed of sound. The factor

$$\Lambda_\ell = \frac{1}{|K_\ell|^2 |k_\ell^2 - k^2|^2}. \tag{5}$$

contains the normalization factor for the mode shapes K_ℓ and the difference between the wavenumber k and the complex eigenvalues

$$k_\ell = (\omega_\ell / c + i\delta_\ell) = ||\mathbf{k}_\ell||_2, \tag{6}$$

where $\mathbf{k}_\ell$ contains the eigenvalues in Cartesian coordinates. For a more detailed derivation of the stochastic modal energy decay model, the reader is referred to [9].

The damping factor δ_ℓ and the decay time of a mode are reciprocally related, i.e. the decay time is the inverse of the damping factor. Hence, a modal reverberation time analogous to the definition by Sabine [10] can be calculated as [11, p. 155]

$$T_{\ell,60} = \frac{6 \ln(10)}{\delta_\ell}. \tag{7}$$

3. Estimation of Absorption

Kuttruff [4] introduced a stochastic model for energy decay in a room, based on the distribution of damping constants of the modes constituting the sound field. Based on theoretical considerations, he showed that the average damping of energy decay curves, i.e. the initial decay rate, corresponds to the energy-weighted mean

$$\bar{\delta} = \frac{\sum_\ell A_\ell \cdot \delta_\ell}{\sum_\ell A_\ell} \tag{8}$$

of the damping distribution $H_\delta(\delta_\ell, A_\ell)$.

Similar to Balint et al. [6], we assume that Sabine's reverberation formula may still be used to estimate the absorption coefficient on a mode-by-mode basis. The band-averaged absorption coefficient can hence be calculated as

$$\bar{\alpha} = \frac{1}{L} \sum_\ell \alpha_\ell = \frac{4V}{cS} \left(\bar{\delta}_o - \bar{\delta}_e\right), \tag{9}$$

where L is an arbitrary normalization factor, and V and S are the volume of the room and the surface area of the test specimen, respectively. The damping factors $\bar{\delta}_o$ and $\bar{\delta}_e$ are the band-averaged damping factors of the occupied and empty room, calculated using Eq. (8). Note that the average damping constants can be used here due to the linearity of Eq. (9).

4. Simulation Setup

We estimate damping distributions for a rectangular room in two conditions: the empty room, with low absorption on all surfaces, and the occupied room, with increased absorption on the floor. The room has dimensions $(L_x, L_y, L_z) = (6.25\,\mathrm{m}, 7.85\,\mathrm{m}, 4.9\,\mathrm{m})$ and an approximate volume of $245\,\mathrm{m}^3$. The sound source is located in a corner close to the ceiling to ensure uniform modal excitation. Note that due to the integration over the entire volume in Eq. (3) no discrete receiver positions are used and the energy density represents perfect averaging over the entire room.

The surface impedance of the absorbing test specimen in the occupied condition was calculated using the Komatsu [12] model combined with the transfer matrix model given by Allard and Atalla [13]. A flow resistivity of $10.5 \times 10^3\,\mathrm{Ns/m}^4$ and a material thickness of $10\,\mathrm{cm}$ were used. The resulting value was additionally scaled by a factor of $\left(\frac{10.8\,\mathrm{m}^2}{L_x L_y}\right)^2$ to avoid non-representative overdamping of modes in the context of the international standard ISO354:2003 [1], which specifies the surface area of the test specimen to be within a range of $10\,\mathrm{m}^2$ to $12\,\mathrm{m}^2$. The eigenvalues in Eq. (6) were calculated numerically using the approach suggested by Nolan and Davy [8]. Due to limitations numerical estimation of eigenvalues only the magnitude of the surface impedance was used, discarding phase information. The damping distributions were calculated for all third octave frequency bands between 80 Hz and 1.250 kHz.

5. Results

5.1. Histogram Analysis

Figure 1 shows the discrete energy-weighted damping distributions estimated for the empty (left column) and occupied (right column) room conditions for the 125 Hz, 250 Hz, 500 Hz and 1000 Hz third-octave frequency bands. While it is evident that the histograms for all bands are multimodal, the number of visually separable groups is lower than the predicted seven by Hunt et al. [3], as groups with similar damping merge. In the occupied condition, two distinct groups

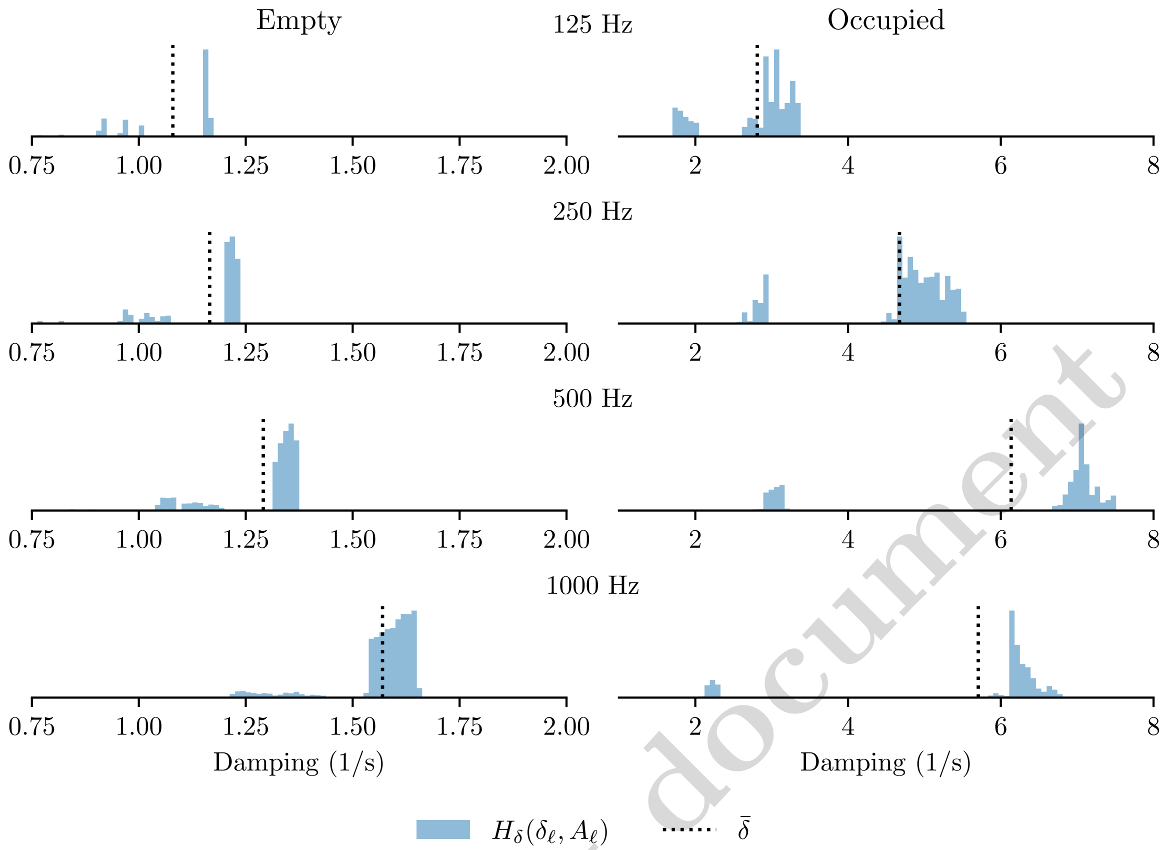

Figure 1: Energy-weighted damping histograms for the empty (left column) and occupied room (right column) for the 125 Hz, 250 Hz, 500 Hz and 1000 Hz third-octave bands (top to bottom). The energy-weighted mean damping constant is indicated by the dashed vertical line.

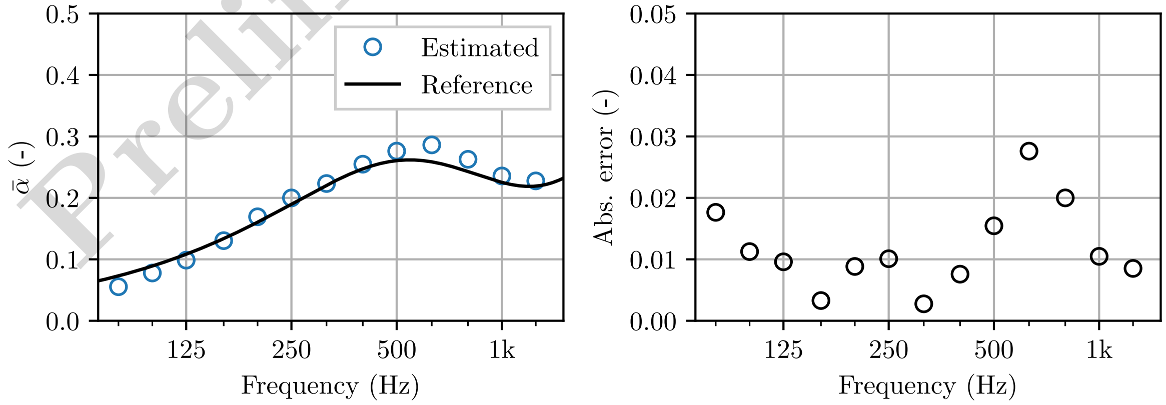

Figure 2: Left: third-octave absorption coefficients estimated using the energy-weighted damping histogram and reference random-incidence absorption coefficients using Paris' law. Right: absolute error between estimation and reference.

are evident. These correspond to groups of tangential and axial modes in the *xy*-plane, which decay at a slower rate (low damping) than the remaining modes (high damping). Due to the large difference in their damping, these two groups result in distinctly multi-exponential energy decay curves. An estimation of the reverberation time using linear regression as per ISO354:2003 [1] would lead to estimation errors as the regression interval is biased towards the damping of the slowly decaying modes. Note, however, that the damping of both groups is higher than in the empty condition, indicating that these modes should not be considered unaffected by the presence of the absorbing test specimen. Interestingly, the energy-weighted damping constant $\bar{\delta}$ does not correspond to the location of the maximum or mean of the highly damped mode group, but is located between the two groups.

5.2. Absorption coefficient

Figure 2 shows the third-octave-band-averaged absorption coefficients estimated using Eq. (9) and the reference random-incidence absorption coefficients calculated using Paris' law and the transfer matrix model by Allard and Atalla [13], as well as the absolute error between both. Overall, good agreement is observed between the absorption coefficient estimated via the energy-weighted average damping and the reference. Overall, the absolute error remains below 0.03 for all frequency bands. Two general trends are observed: firstly, the error increases towards lower frequencies and at frequencies where the specimen shows higher absorption. The increase towards lower frequencies is likely due to a smaller number of oblique and tangential modes compared to the number of oblique modes, thus resulting in less uniform angular distribution of incident energy on the absorbing surface. An explanation for the maximum of the error in the 630 Hz third-octave band is not immediately evident; however, it may be due to the use of Sabine's equation for the estimation of the absorption coefficient.

6. Conclusions

As indicated in Section 5.1, the damping distributions highlight non-uniform damping of the sound field, especially in the occupied condition where two distinct groups of damping constants are observed. The results in Section 5.2 show that the energy-weighted mean damping constant of the occupied and empty room estimates the absorption coefficient of the test specimen accurately, even though the damping distributions yield distinctly multi-exponential energy decay curves. Interestingly, the energy-weighted mean damping constant does not correspond to the maximum or mean of the highly damped mode group, which Balint et al. [6] estimated and used to calculate Sabine's absorption coefficient. Further, the difference in damping for all modes, grazing and non-grazing, between the empty and occupied conditions indicates that the damping of grazing modes should not be neglected when estimating the absorption coefficient. Future work will focus on the estimation of damping distributions from experimental energy decay curves, including comparisons with results obtained using ISO354:2003 [1] and the approach suggested by Balint et al. [6].

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