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SIMULATION OF THE ACOUSTICS IN OPEN-AIR VENUES USING BIDIRECTIONAL SCATTERING

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Abstract

In open spaces such as amphitheaters or open-air concert venues, surprisingly large reverberation times can be found, despite the absence of the ceiling and side walls. Previous research on Greek and Roman theaters showed that reflections between the stage building and the steps in the audience area determine the main components in the impulse responses. So far, such models have been based on random-incidence scattering coefficients, and the results showed that a reverberant field can indeed be simulated in good agreement with measurement results in one-third octave bands. Periodic surface structures such as steps, however, lead to a specific directional scattering pattern with distinct frequency-dependent lobes. With more detailed characterization of this effect, particularly with narrow-band analysis, the reverberant field can be analyzed further. In the presentation, the simulation using the concept of bidirectional scattering will be presented, and the application in the above-mentioned case study.

Keywords: *bidirectional scattering, simulation, 2D reverberation, open-air venues*

1. Introduction

Ancient Greek and Roman theaters exhibit surprisingly large reverberation times despite being open-air structures. Farnetani [1] showed through measurements and scale model experiments that a 2D reverberant field builds up between the stage building and the stepped seating area (*cavea*), and that scattering from the cavea steps is the main source of the late reflected energy. Using boundary element method (BEM) simulations and ray tracing with random-incidence scattering coefficients (RSC), the measured reverberation could be reproduced in one-third octave bands. However, periodic structures such as theater steps produce directional scattering patterns with frequency-dependent lobes that a single random-incidence coefficient cannot capture. We hypothesize that these frequency-dependent lobes, in particular pronounced retroreflection back towards the stage building, are a contributing factor to the surprisingly large reverberation times reported for such venues.

To address this, bidirectional scattering coefficients (BSC) have been introduced [2], which characterize the redistribution of incident sound energy across all outgoing directions. There, the BSC was determined from full 3D BEM simulations. In this contribution, we apply this approach to a theater geometry similar to the one studied by Farnetani and present BSC results for a staircase profile representative of the cavea, and additionally show that the determination can equally be carried out from 2D simulations for surfaces with 1D periodicity, exploiting their translational invariance.

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2. Bidirectional scattering coefficients

The bidirectional scattering coefficient (BSC) describes how incident sound energy from a direction $\mathbf{\Omega}_i$ is redistributed into each outgoing direction $\mathbf{\Omega}_o$ upon reflection at a surface [2]. For a periodic surface with structural wavelength Λ, an incident plane wave generates a discrete set of reflected plane waves (scattering orders) whose directions Φ_n follow from the grating equation [3]

$$\cos\Phi_n = \cos\Phi_0 + \frac{n\lambda}{\Lambda}, \qquad n = 0, \pm1, \pm2, \ldots \quad (1)$$

where Φ_0 is the angle of incidence in the plane perpendicular to the corrugations and λ is the acoustic wavelength. Only orders for which $|\cos\Phi_n| \leq 1$ propagate; the zeroth order (n=0) corresponds to specular reflection.

For an incident wave arriving from an arbitrary direction, the out-of-plane component Θ_0 (angle between the wave vector and the plane perpendicular to the corrugations, Figure 1) effectively scales the wavenumber to $k' = k \cos \Theta_0$, following the projection principle described by Embrechts et al. [3]. This allows the 3D scattering problem to be reduced to a set of 2D problems.

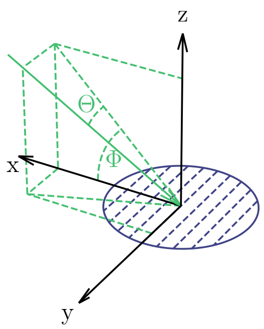

Figure 1: Coordinate system used in this work [3].

2.1. Numerical determination

The BSC of the staircase profile (Figure 2) is determined numerically using BEM in COMSOL Multiphysics. Unlike the full 3D simulations used in [2], the surface and the incoming and outgoing directions are here projected onto the 2D cross-sectional plane perpendicular to the step edges, exploiting the translational invariance of the geometry along the step direction. In this 2D domain, the scattered pressure field of the sample and a flat reference surface of equal length are computed for each combination of incidence angle and frequency, at a fraction of the computational cost of a 3D simulation.

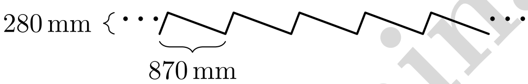

Figure 2: Surface of the steps with an angle of 20°.

The number of structural periods is chosen per incidence angle to ensure convergence of the total reflected energy: for near-grazing incidence ($\Phi_0 < 5°$) up to 160 periods are used, decreasing to 12 periods for $\Phi_0 > 25°$ (Table 1). For this number of repetitions, the error in the total reflected energy is below 10 %. The error is calculated in a manner similar to [3], where the sum of all reflected energy should be 1. The incidence angle Φ_0 is sampled every 10° from 10° to 170°. For the out-of-plane angle, 2D solutions are mapped to 3D via the frequency-scaling relation $f' = f \cos \Theta_0$, with both angles sampled at 10° resolution. The frequency resolution is set to 12 frequencies per octave band, covering the range from 700 Hz to 12 kHz. Below the onset frequency $f_{\text{onset}} = c/(2\Lambda)$, only the specular order propagates and the scattering coefficient is zero. For the staircase profile studied here ($\Lambda = 870$ mm), this onset lies at approximately 197 Hz.

Table 1: Number of structural periods used per incidence angle range, chosen such that the error in the total reflected energy is below 1 %.

Incidence angle Φ_0	Number of periods
$\leq 5°$	160
$\leq 8°$	120
$\leq 10°$	80
$\leq 13°$	60
$\leq 15°$	40
$\leq 25°$	20
$> 25°$	12

Therefore, the scattering order starts with 197 Hz, so the random-incidence scattering coefficient of a periodic surface is theoretically expected to be zero below f_{onset} [3]. The random-incidence scattering coefficient is obtained from the directional values using the Paris formula [2], integrating over all incidence directions.

3. Results

3.1. Random-incidence

Figure 3 compares the random-incidence scattering coefficient obtained by integrating the simulated BSC over all incidence directions with the Paris formula against the free-field and diffuse-field data reported by Farnetani [1]. The diffuse-field values were measured according to ISO 17497-1 [4] in a 1:5 scale-model reverberation room, while the free-field values were measured in an anechoic chamber with an angular resolution of 15°. As expected, the simulated RSC is zero below $f = 197\,Hz$. The sample used by Farnetani [1] contained 9 repetitions for both measurements, whereas the number of repetitions in this work is at least 12 (Table 1). This could explain the deviations between all scattering coefficients up to $f = 1\,kHz$. After that the scattering coefficients are close to each other, which also aligns with the fact that the edge effect is reduced for higher frequencies.

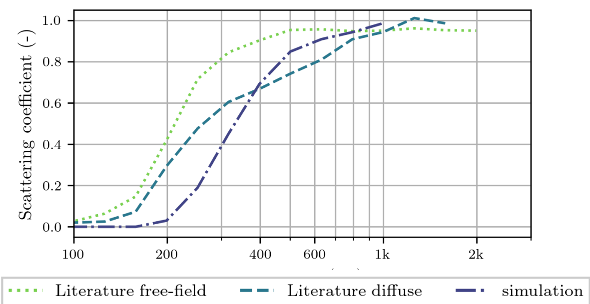

Figure 3: Comparison of the random incidence scattering coefficient from [1] and from simulation.

3.2. Directional scattering

Figure 4 shows the bidirectional scattering coefficients of the steps for an incidence angle of $\Phi_0 = 20°$, chosen

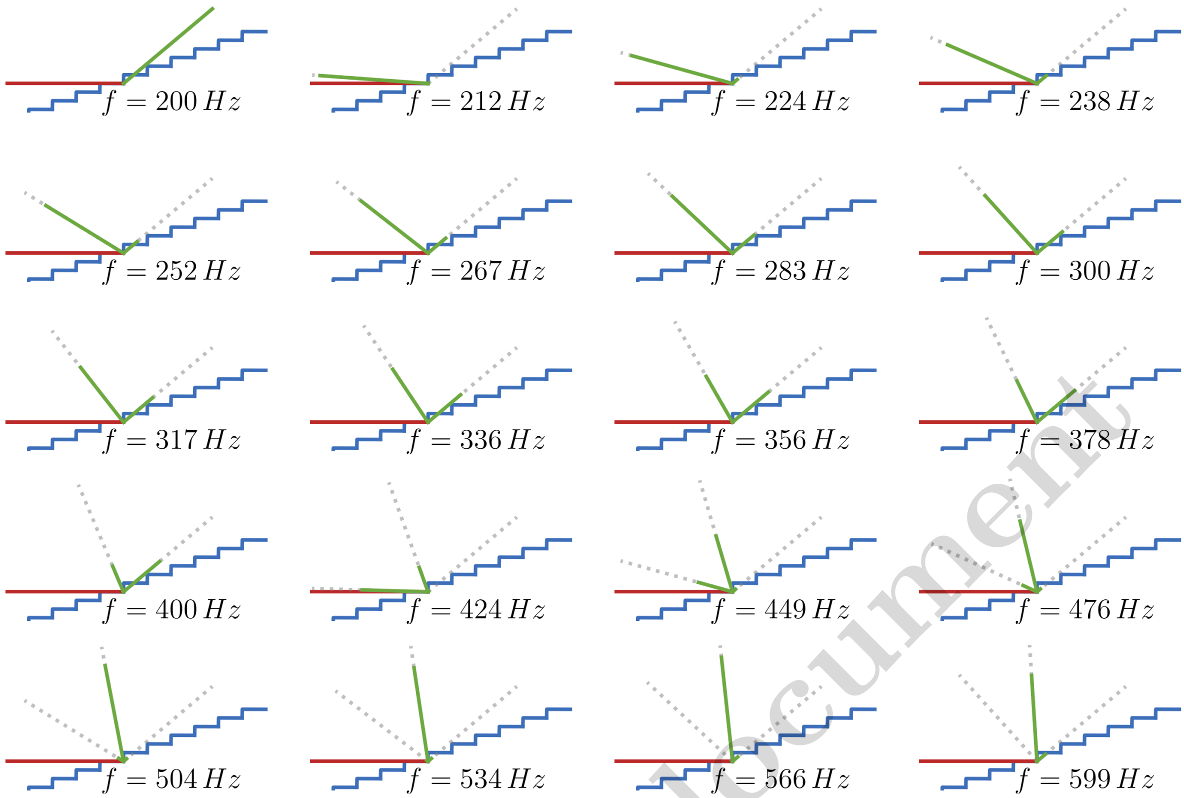

Figure 4: Bidirectional scattering coefficients for an incoming elevation angle of 20° for different real scale one-twelfth octave frequencies ranging from 200 Hz to 600 Hz. This leads to a range of normalized frequencies form $\Lambda/\lambda = 0.5$ to $\Lambda/\lambda = 1.5$.

to represent the reflection path between the steps and the stage building. The dotted gray lines indicate the possible reflection directions, normalized to a length of 1, while the green lines show the fraction of the incident energy scattered into each of these directions. A pronounced retroreflection lobe emerges at certain frequencies (Figure 4 for 212 Hz and 424 Hz), whereas at other frequencies the scattered energy instead remains concentrated towards the top and would therefore leave the theater (Figure 4 for 283 Hz to 400 Hz and for 476 Hz to 599 Hz). This indicates that octave band values alone might be too broad to reproduce the sound field in the theater. A narrow-band analysis is needed to resolve this observation in more detail.

4. Conclusion and Outlook

The bidirectional scattering coefficient of a staircase profile representative of the cavea of ancient open-air theaters was determined numerically using 2.5D BEM simulations and compared to random incidence scattering coefficients measured in diffuse, and free-field [1]. This confirms that, for surfaces with 1D periodicity, the BSC determination method introduced in [2] using full 3D simulations can equally be carried out from computationally cheaper 2D simulations. The random-incidence coefficient derived from these simulations reproduces the theoretically expected onset behavior for a periodic surface, rising from $\Lambda/\lambda = 0.5$, whereas Farnetani's measurements show scattering starting at somewhat lower Λ/λ, likely due to the low number of repetitions of the physical sample. The RSC cannot capture the pronounced, frequency-dependent retroreflection lobes that are characteristic of the periodic step structure. Since these directional effects occur precisely in the reflection path between cavea and stage building that governs the long reverberation times observed in such venues [1], we hypothesize that this retroreflection contributes to the surprisingly large reverberation times reported for these open-air theaters even if they occur only in narrow frequency bands. Confirming this, however, requires propagating the directionally resolved BSC into a full room-acoustic simulation of the reverberant field; this final proof is left to future work outlined below.

As a next step, the BSC data presented here will be converted into a custom bidirectional reflectance distribution function (BRDF) and used in an acoustic radiosity simulation of the full theater geometry with *sparrowpy* [5], [6], allowing a direct, narrow-band comparison of the resulting room-acoustic parameters against Farnetani's small-scale measurements and against simulations based on conventional random-incidence scattering. This narrow-band analysis of the simulated reverberant field is required to finally confirm whether the retroreflection lobes identified here are indeed responsible for the exceptionally long reverberation times observed in open-air theaters and how these excessive reverberation times in narrow bands affect the sound perception.

5. Acknowledgments

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